ONLINE ASSESSMENT: A STUDY OF THE VALIDATION AND IMPLEMENTATION OF A FORMATIVE ONLINE DIAGNOSTIC TOOL IN DEVELOPMENTAL MATHEMATICS FOR COLLEGE STUDENTS

A Dissertation

by

TAUGAMBA KADHI

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2005

Major Subject: Curriculum and Instruction

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ABSTRACT

Online Assessment: A Study of the Validation and Implementation of a Formative Online Diagnostic Tool in Developmental Mathematics for College Students. (August 2005)

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Co-Chairs of Advisory Committee: Dr. Gerald Kulm
Dr. Janie Schielack

This Research and Design (R&D) study models the methodology necessary to replicate an online assessment instrument designed to assess student skills and *facets* of thought while understanding Multiple Meanings and Models of Fractions (MUL) in college level developmental mathematics. The researcher used cognitive research done in the area of fractions to design this instrument that both documents and assesses *facets* of thought or reasoning strategies used by students. The final *facet* cluster is a table that ranks these facets from least to most problematic, documenting the student facets of thought across the content objective MUL.

Over 500 student and teacher participants were used in the design and development of *Fraction Diagnoser*. All participants were affiliated with college developmental mathematics in Texas, representing four colleges and universities. Forty-eight student participants were individually interviewed to ascertain facets of understanding on the topic of MUL. Seven teacher participants were individually interviewed as to the effectiveness of *Fraction*

Diagnoser in the classroom after the final step of the R&D cycle. Content experts were used to design the questions assessing skills and facets.

Fraction Diagnoser was built using the Borg and Gall R&D cycle as its blueprint. Nine of the ten steps of the R&D cycle were used in the development of the instrument, excluding just the final product revision due to cost and time restraints. According to Borg and Gall (1996), a dissertation R&D should be limited to a few steps, but all of the steps used for this R&D allowed for the researcher to completely address all of the research questions. During the steps of the R&D cycle, validation and reliability analyses were done to statistically address the effectiveness of *Fraction Diagnoser*. Final interviews with the teacher participants supported findings in recent research on the effective use of online assessment. Implications for practice and recommendations for further study were also addressed.

DEDICATION

To my grandmother Elnora Jackson Kearse. Undoubtedly, her prayers, guidance, and unconditional love are the main reasons I am who I am today.

ACKNOWLEDGEMENTS

This was the hardest part to write in this entire book and it was done last because there are so many people I would like to thank. It is said that "it takes a village to raise a child", and I believe I am a product of a village. So many friends, family, and people have helped me along the way that it is laughable to even think of listing them all. I am truly appreciative of any acts of kindness and I really would like to thank everyone who has provided any assistance. But I am limited.

I would first like to thank my entire committee for their commitment to work with me. To my Co-Chair Dr. Gerald Kulm, I would like to thank him for giving me an opportunity. I know somewhere five years ago he reviewed my application and decided to give me a chance. For that I will always be indebted. To Dr. Stephanie Knight, I would like to thank her for her timely intervention in my graduate student career. She has since been an unwavering model of knowledge, professionalism, and class. To my surrogate mother Dr. Mary Margaret Capraro, I would like to say that there has been no one at Texas A&M University who has committed more time to both my personal and professional growth. I will work hard to make you proud. And last but definitely not least, my Co-Chair Dr. Janie Schielack. Words cannot express the gratitude I feel toward the leadership, guidance, and motivation she provided in many tough situations. I can honestly say that she properly motivated me to do my best work.

I also would like to thank the professors and mentors who were essential to my success. Dr. Jim Minstrell, the designer of *Diagnoser*, actually sent me all of his research concerning *facets* and was always available to shed more light on the topic. I can't thank him enough for that. Dr. Cathy Loving was my first and last classroom professor, so she definitely deserves a commitment nod. My transcript is full of Dr. Bruce Thompson's classes, and every day with him was an adventure. Dr. Lynn Burlbaw was always full of encouragement and Dr. Carol Stuessy was a joy to learn from. But my most inspirational professor, mentor, and friend was Dr. Robert M. Capraro. While Dr. Capraro is arguably the toughest professor at Texas A&M University, he is unquestionably one of the most dedicated. His commitment to the overall welfare and growth of his students is unparalleled and I look forward to working with him in the future.

And finally, I would like to thank my friends and family that have been crucial to providing the balance necessary for surviving the stress of a dissertation. To my best friends: KJ, Sean, Jory, AC, Charleen, BD James, Jay, Warren, Tenika, David, Marcus, and Neysha, I would like to thank for providing a social outlet. To my family: Kelvin, Scrub (LaTonya), Krystal, Mom, and Dad, I would like to say thanks for being there. Hopefully, this is just the beginning!

I would like to conclude by sharing the phrase that I recited daily to help me stay motivated. The author is unknown, but the saying goes, "Motivation is not an act of spontaneous combustion; you've got to set yourself on fire!" Thank you all for believing in me.

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CHAPTER I

INTRODUCTION

Imagine a mathematics teacher who has just received two exact grades from two different students. Should the teacher assume that both students have the same amount of knowledge or have the same difficulties? Did the students follow the same strategies and are their reasoning and ways of thinking the same? That scenario exemplifies an inherent problem with most summative assessments and their inadequacies in providing insight into student reasoning and thought. These thoughts are keys to identifying and analyzing students' mathematical conceptual understanding.

Assessment, Technology, and Developmental Mathematics

Finding assessment instruments that give useful feedback to the students, teachers, and researchers is a very difficult endeavor. A simple explanation for the difficulty could be that the focus broadens for each group. The student is concerned about the student, the teacher is concerned about the class, and the administration and some researchers are concerned about the population. This disparity, along with other issues, adds to the increasing difficulty to find proper assessment strategies.

This dissertation follows the style of the *Journal for Research in Mathematics Education.*

Regardless of the different foci of the parties involved, knowledge of individual student thought is a primary issue in mathematics education. In fact, the latest educational reform policies in mathematics education request increased attention to “the process” as well as “the answer” (National Council of Teachers of Mathematics (NCTM), 2000). Therefore, traditional summative assessment instruments are less effective in reformed-oriented mathematics classrooms.

Researchers focusing on assessment-centered learning environments contend that students do not always record the procedures they follow to attain their answers (Kulm, 1994; Romberg, 1992). Kulm (1994) stated that this process is a far more difficult one to implement because students are not required to list their thought processes or strategies on most assessments. Consequently, various researchers have suggested providing teachers with knowledge and tools to aid in deducing students’ reasoning (Behr, Harel, Post, & Lesh, 1992; Bransford, Brown, & Cocking, 1999; Minstrell, 2002, 2001, 2000; Research Advisory Committee (RAC), 1998).

In order to assist in the assessment or documentation of student strategies or thoughts, computer-based assessment programs can assist in providing efficient and immediate feedback of student reasoning (Caverly & MacDonald, 2003). Many researchers agree that informational and instructional technologies help to produce a personalized learning environment that can be learning goal specific with reasoning assessed immediately and in real time (MacDonald, 2001; NCTM, 2000; Donovan, Bransford, & Pellingrino, 1999). The computer program *Diagnoser*, based on the research done by Minstrell (2001), assesses and documents students' thinking while providing immediate feedback as they solve physics problems. *Diagnoser* provides a clear example of the efficiency and effectiveness that instructional technologies can provide.

The use of technology in developmental mathematics promises to have a profound effect on student education. According to Caverly and MacDonald (2003), students in developmental mathematics have shown improvement with the use of some instructional and informational technologies. Also, Clements and Battista's (2000) research strongly supports the design of effective software to be used as a cognitive tool to help novices achieve automation of mathematical skills prior to carrying out higher-order thinking. Therefore, theoretically, a replication of Minstrell's (2002) metacognitive instrument *Diagnoser* could be developed in college developmental mathematics to document students' thinking and assess and improve students' levels of understanding (Donovan, Bransford, & Pellingrino, 1999; MacDonald, Vasquez, & Caverly, 2002).

Developmental Mathematics Students and Understanding Fractions

Extensive searches of literature in the area of developmental mathematics students and their understanding of fractions found no sources that could be considered valuable information for this study. However, there is extensive literature on student thought throughout secondary school mathematics on the understanding of rational number ideas and fractions. Specifically, the researchers of the Rational Number Project found and documented that the understanding of fractions and key number concepts are continually problem areas for mathematics students of all ages (Behr, Harel, Post, & Lesh, 1992).

Statement of the Problem

Alternative assessments are always needed. All assessment serves a purpose and the best assessments usually provide information to assist the user in making decisions concerning the student. Although not popular in their high-stakes form, large scale summative assessments give quick and accurate information concerning student achievement. But more information concerning the student is always needed if optimum educational levels are to be reached. Specifically, in the case of developmental mathematics students or students that are diagnosed with learning issues, cognitive research should be used to provide an assessment that focuses on providing insight into the thought processes of these “novice” learners. Because researchers of the Rational Number Project and mathematics educators throughout the country have documented that the understanding of fractions and rational number concepts are continually problem areas with mathematics students of all ages, this study models the design and development of an online assessment instrument for Multiple Representations and Models of Fractions (MUL) (Behr, Harel, Post, & Lesh, 1994; Behr, Lesh, Post, & Silver, 1983; Bright, Behr, Post, & Wachsmuth, 1988; Cramer, 2001;

Cramer, Behr, Post, & Lesh, 1997a, 1997b; Cramer & Post, 1995; Cramer, Post, & Behr, 1989; Cramer, Post, & Currier, 1993; Lesh, Lamon, Gong, & Post, 1992; NCTM, 2000; Post, Cramer, Behr, Lesh, & Harel, 1993).

Purpose of the Study

This study focused on the design and implementation of an online formative assessment tool (henceforth referred to as *Fraction Diagnoser*) that identified students' *facets* of thinking and levels of understanding for multiple representations and models of fractions (MUL). *Fraction Diagnoser* is an online program administered via the web and used by participants on a personal computer. *Fraction Diagnoser* consists of sets of multiple-choice items that assess student thinking or reasoning related to MUL. These items are set up in a corresponding branching structure and are coded at the end of each specific sequence to correlate to a *facet* of thought (see Definitions of Terms). These *facets* are assigned codes according to each path of reasoning, ranking data according to levels of understanding. An *a priori facet* table, based on Minstrell's chapters (2002) outlining the design of *facets*, was used to organize student thought processes and reasoning.

Research Questions

Specifically, the overarching question that this research focused on was: How well could the *Fraction Diagnoser* assess students' depth/level of understanding of MUL as well as support instructional decision-making?

Validation Questions:

- What is the *Facet* Cluster related to multiple meanings and models of fractions (MUL)?
- (Between subjects) How well did the *Fraction Diagnoser* identify distinct levels of understanding of MUL concepts and skills for individual students in developmental mathematics?
- (Within subjects) What kinds of student information did the *Fraction Diagnoser* provide to describe student growth toward mastery in MUL?

Implementation Question:

- How could teachers use the information provided by *Fraction Diagnoser* to make instructional decisions?

Definition of Terms

For the purpose of this research, the terms listed below are defined according to their use in this study:

- Assessment Items – used to define the items that assessed skills used on the *Fraction Diagnoser*

- Developmental mathematics students – college students scoring less than 230 on the Texas Higher Education Assessment (THEA) test (NES, 2003) and required to take supplemental mathematics instruction
- Diagnostic items – items of the *Fraction Diagnoser* related to interview data and the collection of *facets*
- *Facets* – a set of idea units classifying a known content universe (see Figure 1)
- *Facet Cluster* - a set of related facets that explain or interpret a conceptual idea (e.g. Multiple Meanings and Models of Fractions)

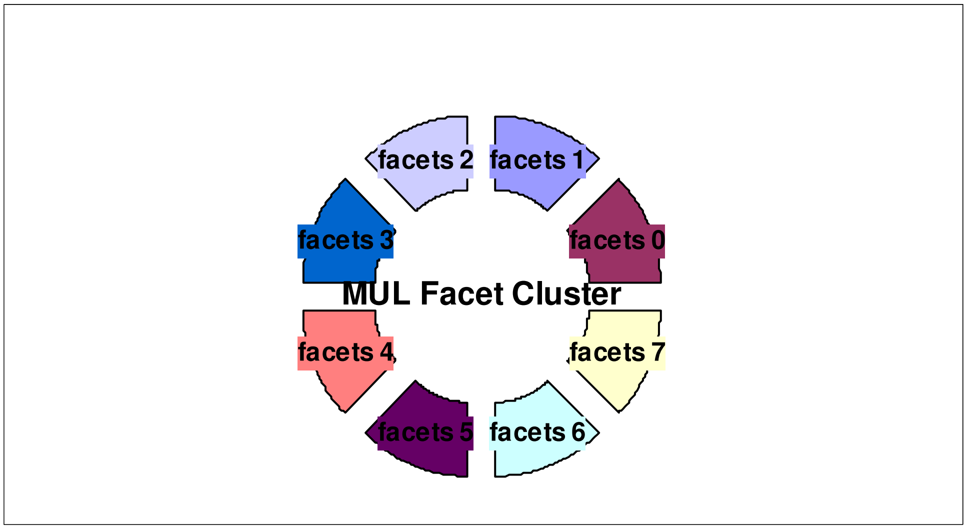


Figure 1. Visual description of relationship between *facets* and *facet* clusters.

- Formative assessment - assessment that is ongoing and provides immediate feedback

- Idea units – observed strategies, knowledge, or skills addressing a specific idea
- Summative assessment - the type of assessment giving a "summary" of student understanding as measured by a test at the end of a unit or cycle

Summary of Introduction

When we know student reasoning or thinking, we can design curriculum, assessment items, and teaching strategies to elicit these ideas, challenge, and guide them (Cobb, 2000; Minstrell, 2002, 2001). Elaboration activities offer an opportunity to test the reliability and validity of new assessment techniques and to explore contexts of application of the new strategies. Assessment embedded in instruction allows students to check on their understanding and allows teachers to monitor progress and identify instructional needs of individuals or of the whole class (Donovan, Bransford, & Pellingrino, 1999; Fuchs & Fuchs, 1986; Kulm, 1994; Minstrell, 2001; NCTM, 2000). Conclusively, in addition to the benefits of analysis of student thinking, this research will also use technology to efficiently offer empirical data to support the body of metacognitive research (Cobb, 2000).

CHAPTER II

LITERATURE REVIEW

Introduction

As stated in the previous chapter, the replicated *Diagnoser*-like program for this study focused on assessing and documenting student thinking processes while students encountered multiple representations of rational numbers. To build a sufficient research base for this study, an intensive review of literature evaluating the research of the University of Minnesota's Rational Number Project (RNP), educational technology advancements in the classroom, and Minstrell's *Diagnoser* research (2002, 2001, 1989) was done.

Although the population for this study is developmental mathematics students in Texas, a literature search on the combined topics of developmental mathematics, technology, and fractions, yielded no hits on several online databases. In fact, research in developmental mathematics is anemic at best. But, research done by the National Evaluation Systems (NES) (2003) concerning developmental students in mathematics indicates that these students lack achievement levels of college algebra students and, therefore, studies by RNP concerning students in secondary mathematics are a suitable research base for developmental students learning fractions. Also, technology in the classroom in developmental education was reviewed. but a sufficient research base could not be found in these studies alone. Therefore, a broader research base of technology was done.

Rational Number Project (RNP)

The Research Number Project (RNP) was the longest lasting federally funded cooperative multi-university research project in the history of mathematics education. The National Science Foundation (NSF) continuously funded the project from 1979-2002 (with the exception of 1983). When the project officially ended in August 2002, the group accumulated each of the publications in their original format and put them in one place for use by present and future researchers and practitioners interested in the study of public school mathematics. Studies of teaching, learning, and assessment are documented throughout the project's research (Cramer, 2001; Cramer, Behr, Post, & Lesh, 1997a, 1997b; Cramer & Henry, 2002; Harel, Post, & Behr, 1988a, 1988b; Post, Behr, & Lesh, 1988; Post, Cramer, Harel, Kiernen, & Lesh, 1998).

Rational Number Project (RNP) research has provided the data for over 85 papers, book chapters, several books and other project publications. The majority of those publications specifically addressed the learning and teaching of rational number concepts including fraction, decimal, ratio, indicated division, measure and operator. Because these studies led naturally to investigations of proportionality with specific attention to the components of proportional reasoning, they also examined the contributions of multiplication and division understandings to rational number concepts. Furthermore, they offered research-based information to improve the designs of effective professional development programs for teachers while concurrently providing appropriate assessment practices in the field of mathematics (Cramer & Henry, 2002). Because studies show that levels of student achievement on standardized tests of rational numbers in developmental mathematics are normally lower than those for the average college student, the studies done by RNP in the secondary schools provide a sound research base for student cognition in developmental mathematics (Caverly & MacDonald, 2003; MacDonald, Vasquez, & Caverly, 2002; Kennedy, 2000).

Rational Number Concepts

Rational-number concepts are among the most difficult and yet important mathematical ideas students of all ages encounter during their school years. The importance can be seen from a variety of perspectives. Specifically, Behr, Lesh, Post, and Silver (1983) found the most important perspectives were:

- practically, where dealing effectively with these concepts vastly improved one's ability to understand and handle situations and problems in the real world;
- psychologically, where rational numbers provided a rich arena in which students developed and expanded the mental structures necessary for continued intellectual development; and
- mathematically, where strong understandings provided the foundation upon which elementary algebraic operations could later be based.

The National Assessment of Educational Progress (NAEP) results have shown that most mathematics students experience significant difficulty learning and applying rational number concepts (Carpenter, Coburn, Reys, & Wilson, 1978; Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980). Even more specifically, results from NAEP indicated that most 13- and 17-year-olds tested could successfully add fractions with like denominators, but only one-third of the 13-year-olds and two-thirds of the 17- year-olds could correctly add 1/2 + 1/3. NAEP findings are consistent with many other studies, indicating a generally low performance on rational-number computation and problem solving (Bart, Post, Behr, & Lesh, 1994; Behr, Lesh, Post, & Silver, 1983; Cramer, 2001; Cramer, Behr, Post, Lesh, 1997a; Cramer & Henry, 2002). Low levels of performance may surprise some, because most school programs tend to emphasize procedural skills and computational algorithms for rational numbers. Some theories cite poor performance as a direct result of curricular emphasis on procedures rather than the careful development of important functional understandings (AAAS, 1998; Behr, Lesh, Post, & Silver, 1983; Lesh, Hoover, Hole, Kelly, & Post, 2000; NCTM, 2000).

Rational Number Project (RNP) researchers also found that many problem areas in school mathematics are related to rational number ideas (Behr, Lesh, Post, & Silver, 1983; Lesh, Hoover, Hole, Kelly, & Post, 2000). Developing rational number ideas has consistently been an ideal context in which to investigate general mathematical concept acquisition. The RNP researchers cited findings such as: (1) much of the development occurs on the threshold of a significant period of cognitive reorganization (that is, the transition from concrete to formal operational thinking), (2) interesting qualitative transitions occurred not only in the structure of the underlying concepts but also in the representational systems used to describe and model these structures, (3) the roles of representational systems are quite differentiated and interact in psychologically interesting ways because both figurative and operational task characteristics are critical, and (4) the rational number concept involves a rich set of integrated subconstructs and processes, related to a wide range of elementary but deep concepts (e.g., measurement, probability, coordinate systems, graphing, etc.), clearly exhibiting why rational numbers should always be a focus of study.

Some of the cognitive research in the RNP was based on Piaget's (1965) operational aspects of tasks and concepts studies. He used the term *horizontal decalage* to explain the fact that, whereas it may be useful to think of a person as being characterized by a given cognitive structure, that person will not necessarily be able to perform within that structure for all tasks. This theory was used in the RNP to address why it is common to encounter *horizontal decalage* with respect to rational number concepts, in that models embodying the same concept vary radically in the ease with which mathematics students understand them. Therefore, information about how task variables influence task difficulty is important for those who must select or devise appropriate models to illustrate rational number concepts (Behr, Lesh, Post, & Silver, 1983; Harel, Behr, Post, & Lesh, 1987; Lesh, Hoover, Hole, Kelly, & Post, 2000).

Several long-term teaching experiments in the RNP studies concerning the teaching and learning of fractions among students, led to the development of a curriculum (Bezuk & Cramer 1989; Cramer, Behr, Post, & Lesh, 1997a, 1997b; Cramer & Henry, 2002; Post, Wachsmuth, Lesh, & Behr, 1985). This curriculum was created for these teaching experiments and then revised so that the basis of what was learned reflected the following four beliefs: (1) mathematics students' learning of fractions could be optimized through active involvement with multiple concrete models, (2) most mathematics students needed to use concrete models over extended periods of time in order to develop mental images needed to think conceptually about fractions, (3) mathematics students benefited from opportunities to talk to one another and with their teacher about fraction ideas as they constructed their own understandings of fraction as a number, and (4) teaching materials for fractions would focus on the development of conceptual knowledge prior to formal work with symbols and algorithms.

After a decade of research on the teaching and learning of fractions among those students, Cramer and Henry (2002) found that of the four pedagogical beliefs listed above, the second was the most important. They found that in order to develop fraction sense, most of the students needed extended periods of time with physical models such as fraction circles, Cuisenaire™ rods, paper folding, and chips. Those models allowed students to develop mental images for fractions, and those mental images enabled students to understand more about relative fraction size. Students used their understanding of fraction size to operate on fractions in a meaningful way, and multiple models of fractions were used in all of these RNP teaching experiments. Specifically, the fraction circle model used in combination with the RNP activities (see Figure 2) was reported as the most powerful of the models. Students, while interviewed, consistently referred to fraction circles as the model that helped them order fractions and estimate the reasonableness of various fraction operations (Cramer & Henry, 2002).

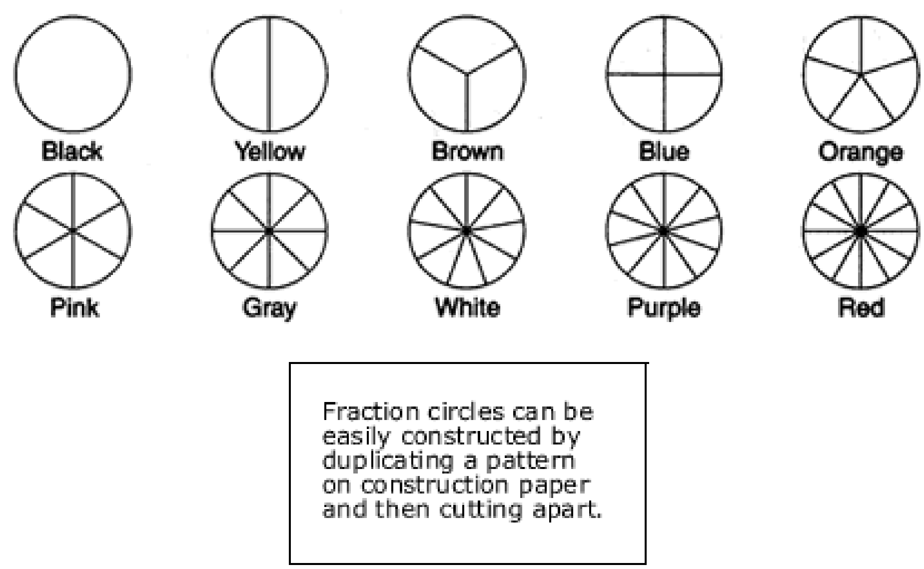


Figure 2. Fraction circles used in the RNP teaching experiments

Various Representations of Rational Numbers

Kieren (1981) provided a research base for a considerable amount of work done by RNP. This researcher identified and discussed five *faces* of mathematical knowledge building used with RNP, relating to the mathematical, visual, developmental, constructive, and symbolic nature of mathematics, learning mechanisms, and learners. Also evaluated were four mathematical subconstructs of rational numbers that led to other studies--*measure, quotient, ratio,* and *operator*—with each providing quantitative and relational rational number experience. Equivalence and partitioning were constructive mechanisms operating across the four subconstructs to extend images and build mathematical ideas.

The Part-Whole or Measure Subconstructs

A part-whole understanding of rational number depends directly on the ability to partition either a continuous quantity or a set of discrete objects into equal-sized subparts or sets. According to Kieren (1981), this subconstruct was fundamental to all later interpretations and was considered to be the fundamental language-generating construct.

It was found that this interpretation is usually introduced very early in the school curriculum, because mathematics students in earlier grades had primitive understandings of the meaning of one-half and the basic partitioning process (Behr et al., 1983; Behr & Post, 1992). Later, the fraction concept was treated in a systematic fashion. Students normally explore and extend rational number ideas through middle school mathematics, and then these understandings are applied in elementary algebra. Numerous researchers found that many students' difficulties in algebra can be traced back to an incomplete understanding of earlier fraction ideas (Behr, Harel, Post, & Lesh, 1993; Behr, Wachsmuth, & Post, 1985; Carpenter, Coburn, Reys, & Wilson, 1978; Carpenter, Corbitt, Kepner, Linquist, & Reys, 1980).

Geometric regions, sets of discrete objects, and the number line were the models most commonly used to represent fractions in the elementary and middle school levels. It was found that interpretations of geometric regions usually display an understanding of the notion of area (Behr & Post, 1981; Behr, Wachsmuth, & Post, 1984, 1985, 1988). Cramer and Henry (2002) cited the 1980 articles by Owens and Sambo and their examination of the relationship between a student's concept of area and her or his ability to learn fraction concepts. They said that Owens found a positive relationship between success on area tasks and success in an instructional unit based on geometric regions, while Sambo reported that deliberate teaching for transfer from area tasks helped students to learn fraction concepts when geometric regions and measurement interpretations were involved.

In another study by Heller, Post, Behr, and Lesh (1990), evidence was found to suggest that the fraction concepts should be introduced using a single model. They also found that the part-whole measurement model was the most natural for young mathematics students and the most useful for addition of like fractions. The Initial Fraction Sequence (IFS), an instructional sequence based on the 1978 research of Ellerbruch and Payne, was cited in this study and emphasized the importance of developing a firm foundation of fraction concepts before introducing mathematics students to operations, relations, or multiple representations on rational numbers. IFS uses rectangular regions because of the ease of making models from strips of paper, and proceeds carefully from concrete or pictorial models to oral fraction names, to natural language written names (e.g., three-fourths), and finally to formal mathematical symbols.

The number-line model adds an attribute not present in region or set models, particularly when a number line of more than one unit long is used. Lesh and Lamon (1992a, 1992b) presented seventh-grade mathematics students with tasks involving the location of fractions on number lines that were one or two units long and for which the number of segments in each unit segment equaled or was twice the denominator of the fraction. Results of the study indicated that, among seventh-graders, associating proper fractions with points was significantly easier on number lines of length one - and when the number of segments equaled the denominator. The findings suggest an apparent difficulty in perception of the unit of reference: when a number line of length two units was involved, almost 25% of the sample used the whole line as the unit. The data also indicated that these mathematics students did not associate the rational number *one-third* with a point for which partitioning suggests *two-sixths*. Such results suggest an imprecise and inflexible notion of fraction among these students.

Behr, Harel, Post, and Lesh (1992) investigated whether or not the type of embodiment (continuous quantity versus discrete quantity) demanded different types of cognitive structures. They asked whether the part-whole interpretations given by Piaget, Inhelder, and Szeminska (1960) were appropriate for both the discrete and the continuous cases of length and area. They required students to divide a quantity equally and completely among a number of stuffed animals. They found that students performed considerably better on tasks involving the discrete case (set-subset) than the continuous case. It was explained that solutions of the continuous quantity tasks (Piaget *et al*., 1960) required a well-developed anticipatory scheme, whereas simply partitioning can be used to solve discrete quantity tasks. In a similar study, it was found that discrete tasks can be solved without treating the set as a whole and without anticipating the final solution (Behr et al., 1983). Because the strategies employed by students for discrete quantity tasks were so different from those employed for continuous quantity tasks, it was assumed and other research similarly concluded that cognitive structures involved in solving rational number problems referring to a discrete model are different from those involved in solving rational number problems referring to a continuous model (Behr, Harel, Post, & Lesh, 1993; Carpenter et al., 1978; Carpenter et al., 1980; Lacampagne, Post, Harel, & Behr, 1988; Lesh & Kelly, 2000; Post, 1989; Post, Behr, & Lesh, 1988).

Rational Number as Ratio

Ratio is a relation that conveys the notion of relative magnitude; therefore, it is considered more appropriately as a comparative index than as a number. When two ratios are equal they are said to be in proportion with one another. Therefore, a proportion is simply a statement equating two ratios. The use of proportions is a very powerful problem-solving tool in a variety of physical situations and problem settings that require comparisons of magnitudes (Cramer, 2001; Cramer, Post, & Behr, 1989; Cramer & Post, 1993a).

The Noelting (1980) study is referenced throughout RNP when researchers were investigating subjects' ability to compare ratios (Behr & Post, 1988; Cramer, Behr, & Bezuk, 1989; Cramer, Behr, Post, & Lesh, 1997a; Cramer & Post, 1995; Cramer & Post, 1993b). Noelting's tasks asked students to specify which of two mixtures of orange juice and water would taste stronger or more “orangy." Three levels were observed among subject responses, ranging from making judgments based only on comparisons of terms, to comparing ordered pairs using multiplicative rules, to the final level of which ordered pairs were seen as belonging to a class.

The use of glasses of water and orange juice suggests a discrete model, but these and many other studies were based on the use of continuous model quantities (Behr & Harel, 1990; Behr & Post, 1988; Cramer, Behr, & Bezuk, 1989; Cramer, Behr, Post, & Lesh, 1997a; Harel, Behr, Post, & Lesh, 1994b; Kieren, 1988; Kieren, Nelson, & Smith, 1985; Orton, Post, Behr, Cramer, Harel, & Lesh, 1995; Post, Behr, & Lesh, 1986). During these studies subjects were asked to find an unknown component of a proportionality statement by equating two ratios involving length, distance, or volume. Researchers identified various levels of cognitive functioning, ranging from random guessing to additive (rather than multiplicative) reasoning, to the most advanced stage at which the data was utilized at a formal level of multiplicative ratio-type thinking.

Rational Numbers as Indicated Division or as Elements of a Quotient Field

According to the part-whole interpretation of rational numbers, the symbol *a/b* usually refers to a fractional part of a single quantity. In the ratio interpretation of rational numbers, the symbol *a/b* refers to a relationship between *two* quantities, but the symbol *a/b* may also be used to refer to an operation. That is, *a/b* is sometimes used as a way of writing $a \div b$. This is the *indicated division* (or *indicated quotient*) interpretation of rational numbers (Cramer & Post, 1995; Post et al., 1998).

Consideration of rational numbers as quotients involves at least two levels of sophistication (Heller, Post, Behr, & Lesh, 1990; Kieren, 1981, 1988; Reiss et al., 1988). On the one hand, 8/4 or 2/3 interpreted as an indicated division results in establishing the equivalence of 8/4 and 2, or 2/3 and $.\overline{6}$. But rational numbers can also be considered as elements of a quotient field, and, as such, can be used to define equivalence, addition, multiplication, and other properties from a purely deductive perspective; all algorithms are derivable from equations via the field properties (Heller, Post, & Behr, 1985; Kieren & Southwell, 1979). Research indicates that this level of sophistication generally requires intellectual structures not available to middle school students because it relates rational numbers to abstract algebraic systems (Harel, Behr, Post & Lesh, 1994a; Kieren, Nelson, & Smith, 1985; Reiss et al., 1988, 1985;).

Rational Number as Operator

In the Cramer (2001) study the subconstruct of rational number as *operator* imposed on a rational number *p/q* an algebraic interpretation; *p/q* was thought of as a function that transformed geometric figures to similar geometric figures *p/q* times as big, or as a function that transformed a set into another set with *p/q* times as many elements. Other studies explained that when operating on a continuous object (length), we think of *p/q* as a stretcher-shrinker combination (Behr, Harel, Post, & Lesh, 1994, 1992, 1991; Kieren, 1988; Cramer & Henry, 2002). Any line segment of length *L* operated on by *p/q* was stretched to *p* times its length and then shrunk by a factor of *q*. A multiplier-divider interpretation was given to *p/q* when it operated on a discrete set. The rational number *p/q* transformed a set with *n* elements to a set with *np* elements and then this number was reduced to *np/q*.

Some research has cited that this rational-number concept can be embodied in a function machine in which *p/q* is thought of as a "*p* for *q*" machine (Behr, Lesh, Post, & Silver, 1983; Harel & Behr, 1995; Noelting, 1980). Thus, 3/5 is thought of as a 3 for a 5 machine: an input of length or cardinality 5 produces an output of length of cardinality 3.

The operator interpretation of rational number was particularly referenced by researchers concerned with the equivalence of fractions and the operation of multiplication (Behr, Khoury, Harel, Post, & Lesh, 1997). They found that the problem of finding fractions equivalent to a given fraction was similar to that of finding function machines that accomplished the same input-output transformations because the multiplication of fractions involves composition of functions.

Also, the studies conducted by Kieren (1988, 1981, 1976) and his colleagues (Kieren & Nelson, 1981; Kieren, Nelson, & Smith, 1985; Kieren & Southwell, 1979) and Noelting (1980) have investigated the stage development of the operator and ratio constructs and the relationship between them in students' thinking. Other research by the RNP found that further analysis of students' descriptions of how a machine works indicated that students thought subtraction and not multiplication (Bart et al., 1994; Behr, Harel, Post, & Lesh, 1994, 1992; Behr, Post, & Lesh, 1981; Cramer & Post, 1995, 1993a, 1993b). According to Cramer & Post (1993b), this was particularly true for the younger students. A second important finding was the role one-half played in subjects' thinking; 91% of the subjects mastered the "one-half" tasks. Even students who knew that a machine was not a one-half machine would give a one-half response when confused. Apparently the students' higher rate of success on one-half tasks, and greater familiarity with the number itself, led to misapplications and misconceptions when asked to properly apply the rule. In one of the earlier studies this type of error was made by 47% of the subjects (Kieren & Nelson, 1978).

The Behr, Harel, Post, and Lesh (1993) study examined differences between a student's ability to perform operator tasks when the task was embedded in a function machine compared with a "simpler" approach consisting of patterns of symbolic input-output number pairs. Their analysis of variance of correct responses indicated no significant differences due to representation mode. Three levels of rational number operator development were observed in data from both types of tasks. The authors suggested that understanding of equivalence class and partitioning were the important mechanisms underlying this development. “Partitioning” was defined as the division of a set into subsets. Applying equivalence class thinking to a one-third task, a subject who correctly pairs 2 with 6 explained it as "divided by 3," but a more sophisticated use of the mechanism was required for success on the non-unit fraction of "two-thirds;" to pair 90 with 60, a student would have to think, "divide by 3 and take 2 of them". The general fractional operator appeared to require the coordination of the partitioning of two subsets of numbers with a multiplicative operation, in this case doubling. Subjects in the machine representation condition used this dual step partitioning strategy most often. In the pattern representation condition, they found that a pattern explanation frequently accompanied a correct response. Generally, in pairing 24 with 16 in the two-thirds task, the authors reported comments such as, "Well, I know 12 went to 8 so I just doubled to get 16" (as cited from Behr et al., 1993). Findings noted a higher level of performance was observed in the machine group at a younger age compared with the pattern group.

In addition, another study gave a single group of subjects both operator tasks and orange juice tasks. It was concluded that *(a)* there was an indication that students who were able to partition are also able to perform comparisons and to recognize equivalences, and *(b)* the level of cognitive thinking necessary for successful performance on general operator tasks was relatively the same level as that needed for successful performance on the multiplicative-equivalence comparisons in the ratio tasks (Behr, Harel, Post, & Lesh, 1997).

Proportional Reasoning

Various studies have defined proportional reasoning as the ability to involve an understanding of the mathematical relationships embedded in proportional situations (Bart et al., 1994; Behr et al., 1992; Cramer & Post, 1993a; Harel, Behr, Post, & Lesh, 1992; Post et al., 1998; Tournaire & Pulos, 1985). These relationships are always multiplicative in nature. When looked at algebraically, proportional relationships are expressed with the form of $y = mx$, again emphasizing the multiplicative relationship that is inherent in all proportional situations. When seen graphically, a straight line that passes through the origin depicts the proportional situations. From algebra, the m in the equation $y = mx$ designates the slope of the line. This slope m is also the unit rate and the constant factor that relates quantities between two measure spaces (Bart et al., 1994; Post et al., 1998; Tournaire & Pulos, 1985). Cramer (1993a) further explained that all rate pairs for that situation appear on the line $y = mx$.

Furthermore, Cramer and Post (1993a, 1993b) investigated proportional reasoning and the ability to solve a variety of problem types. Specifically, missing value problems, numerical comparison problems, and two types of qualitative situations were among the types of problems that they cited as important for students to understand. They defined proportional reasoning as involving the ability to discriminate proportional from non-proportional situations; researchers found that a proportional reasoner ultimately was not influenced by context nor numerical complexity. Ultimately, these students' ability allowed them to overcome the effects of unfamiliar settings and cumbersome numbers (Cramer et al., 1993a, 1993b; Post et al., 1998). But the understandings underlying proportional reasoning are complex, and research has documented that this type of reasoning develops slowly over several years (Behr, Wachsmuth, Post, & Lesh, 1984; Cramer et al., 1993a, 1993b; Tournaire & Pulos, 1985).

In another study, the Rational Number Project (RNP) administered a survey of proportional reasoning tasks to over 900 seventh- and eighth-grade students (Cramer, Post, & Currier, 1993). The questions included missing value problems, numerical comparison problems, and two types of qualitative problems. Both integer and non-integer relationships were tested. The study varied the contexts in which the problems were given but kept the numerical properties the same across the contexts. The contexts were speed, buying, density, and scaling. The researchers predicted that the more familiar buying and speed contexts would be easier than the less familiar density and scaling contexts (Cramer, Post, & Currier, 1993).

Although the seventh-grade students had no prior instruction in the standard procedure for solving a proportion (cross-multiply and divide), eighth graders had received such instruction a few weeks prior to the survey, hence the much larger incidence of the standard algorithmic approach with missing value problems for eighth graders. Furthermore, eighth-grade students performed better on both the missing value and numerical comparison problems. Other results, not given in the table (See Table 1), show that there was almost no difference (5%) between the groups on the qualitative questions, an area for which neither group received specific instructions (Cramer, Post, & Currier, 1993).

As in other studies, the *unit rate* approach was the most popular strategy and the tactic that was responsible for the most correct answers (Bart et al., 1994; Heller, Post, Behr, & Lesh, 1990; Hoffer, 1988; Post et al., 1988). This approach was characterized by the finding of the multiplicative relationship between measure spaces. Specifically, the unit rate was found through division. For example, if 3 apples cost 60 cents, in order to find the cost of 6 apples the cost for 1 apple is found first by dividing: 60 cents ÷ 3 apples = 20 cents per apple. This unit rate is the constant factor that relates the apples and cost. To find the cost of 6 apples, one simply multiplies 6 apples by 20 cents per apple. This method was very popular with seventh-grade students who were uninstructed in the usual cross-multiply and divide algorithm. Bart, Post, Behr, and Lesh (1994) stated that this result should not be surprising because people make purchases and have had the opportunity to calculate unit prices and other unit rates. Therefore it seemed like a natural way for them to approach those problems.

Other studies also evaluated students responding using the *factor of change* method (Cramer et al. 1993a, 1993b; Post, Cramer, Harel, Kiernen, & Lesh, 1998). The method would be as follows: If I want twice as many apples, then the cost will be twice as much. The *factor of change* method was described as a "How many times greater" approach and was equivalent to finding the multiplicative relationship within a measure space.

Furthermore, using the data some researchers reported a small number of seventh-grade students employing what was called a fraction strategy. This strategy was reportedly used by a much larger percentage of eighth-grade students. The fraction strategy was applied devoid of the problem context (Cramer et al., 1993; Harel et al., 1994; Post et al., 1998). Specifically, rate pairs were treated as fractions by disregarding the labels. In the study, students using this strategy would calculate answers, applying the multiplication rule for generating equivalent fractions as follows:

$$\text{If} \quad \frac{3}{60} = \frac{6}{?} \quad \text{Then} \quad \frac{3}{60} \times \frac{2}{2} = \frac{6}{120}$$

Also found was the fact that the percentage of correct responses for the single item that did not use integral multiples was significantly lower than problems with integer relationships. In this one particular problem, one number of a rate pair was 1.5 times the other. Although this was not a particularly dramatic change, differences in the results were dramatic.

Consequently, the presence of a non-integer relationship did two things: (1) it significantly decreased the level of student achievement and (2) it changed the way in which students thought about a problem (Post et al., 1998). This is evidenced by the significantly lower percentage of students who used the unit rate and factor methods.

Data in Table 1 show that large percentages of seventh and eighth graders in the study were unable to solve these problems. Data not reported here, and also supported by other research, suggest that scaling problems are significantly more difficult than the buying, speed, or density problems (Heller, Post, & Behr, 1985; Heller, Post, Behr, & Lesh, 1990).

Table 1. Percentages of Correct Solutions by Strategies for Missing Value (MV) and Numerical Comparison (NC) Word Problems

	Seventh grade n = 421		Eighth grade n = 492	
Strategy	MV	NC	MV	NC
Unit Rate	28* (15)**	48(26)	14(6)	30 (18)
Factor	17 (5)	12 (5)	7 (3)	8(2)
Algorithm	3 (4)	1 (1)	33 (45)	10 (15)
Fraction	2 (1)	8 (10)	12 (2)	26(25)
Incorrect	50 (75)	31 (57)	35 (44)	32(40)

*The first entry is an average of 3 problems - all numerical values are integral multiples of one another.
**The entry in parentheses is an average for single problems whose numerical values are not integral multiples of one another (Cramer and Post, 1993a, p.405)

Additional Rational Number Subconstructs

An evaluation of the components of the concept of rational number suggests complete comprehension of rational numbers is a formidable learning task. Researchers in RNP found that rational numbers could be interpreted in several ways (referred to as *subconstructs*): a part-to-whole comparison, a decimal, a ratio, an indicated division (quotient), an operator, and a measure of continuous or discrete quantities, contending that a complete understanding of rational numbers required not only an understanding of the subconstructs but also their interrelation. Also, theoretical analyses and empirical evidence suggested that different cognitive structures might have been necessary for dealing with the various rational number subconstructs (Kieren & Nelson, 1981; Kieren & Southwell, 1979; Novillis, 1976; Riess, Behr, Lesh, & Post, 1988; 1985).

Various studies have identified stages in a student's rational number thinking by examining the gradual differentiation and progressive integration of separate subconstructs (Cramer, 2001; Post, Behr, & Lesh, 1986, 1982; Post & Cramer, 1987; Wachsmuth, Behr, & Post, 1983). One important aspect of these studies has been to observe whether or not participants performing at a given stage on tasks involving one subconstruct perform at a comparable level on tasks involving a different subconstruct. Also, relationships between specific skills and certain basic rational number understandings were investigated.

Various researchers in RNP even redefined some of Kieren's (1976) categories and subdivided a couple of others (Behr et al., 1983; Behr, Harel, Post, & Lesh, 1991; Behr & Post, 1992; Behr, Khoury, Harel, Post, & Lesh, 1997; Behr et al., 1994, 1993; Bright, Behr, Post, & Wachsmuth, 1988). Throughout the project they divided fraction understanding into the following seven subconstructs. First, the fractional measure subconstruct of rational number represented a reconceptualization of the part-whole notion of fraction (Behr et al., 1983). It addressed the question of how much there was of a quantity relative to a specified unit of that quantity. Then, the ratio subconstruct of rational number expressed a relationship between two quantities, for example, a relationship between the number of apples and oranges in a fruit basket (Behr et al., 1991). Thirdly, the rate subconstruct of rational number defined a new quantity as a relationship between two other quantities (Behr et al., 1993). For example, velocity was defined as a relationship between distance and time. They also observed that although one added rates in such a context as computing average speed, one seldom adds ratios. Fourth, the quotient subconstruct of rational number interpreted a rational number as a quotient. That is, *a/b* was seen as *a* divided by *b*. In a curricular context this subconstruct was exemplified by the following problem situation:

There are 5 pieces of gum and 2 people. If the gum is shared equally by the two people, how much gum does each person get?

Fifth, a linear coordinate subconstruct of rational number was similar to Kieren's (1976) notion of a measure interpretation. It emphasized properties associated with the metric topology of the rational number line such as betweenness, density, distance, and (non)completeness (Behr et al., 1983). When rational numbers were interpreted as points on a number line, it emphasized that the rational numbers were a subset of the real numbers. Sixth, the decimal subconstruct of rational number emphasized properties associated with the base-ten number system (Behr & Post, 1992). And lastly, the operator subconstruct of rational number imposed on rational number a function concept. A rational number was now a transformation. The stretcher-shrinker notions developed by the University of Illinois Committee on School Mathematics (UICSM) and the Comprehensive School Mathematics Project (CSMP) also represented physical embodiments of this construct (Behr et al., 1997).

There were still no solid answers regarding which of those subconstructs might best serve to develop in mathematics students the basic fraction concept, whether relations on rational numbers, operations with rational numbers, or applications of rational numbers. In the end, it seemed plausible that the part-whole subconstruct, based both on continuous and discrete quantities, represented a fundamental construct for all rational-number concept development. And, it was this point of departure for instruction that many researchers suggested first before involving other subconstructs (Behr et al.,1983; Kieren & Nelson, 1978; Kieren & Southwell, 1979; Novillis, 1976).

Educational Technology as Instructional Technology

It has been often stated that technologies, for decades, had been introduced as having groundbreaking benefits to American education, but despite the optimism, early research found that their use in schools led to a persistent cycle of inappropriate use followed by disappointment and then abandonment (Cuban, 1986). The American Association for the Advancement of Science (AAAS) said the main reason for early failures had been that instructional "innovations" that use new technologies were focused on the lure of the new hardware and its ability to process or deliver information faster, in greater quantities, and from greater distances, rather than on the quality of instruction that the hardware carries or supports (AAAS, 1998). Basically, these hardware-driven, rather than content- or instruction-driven, reforms are what failed.

AAAS and other researchers believed that the hardware-driven reforms' demise could be traced to three major reasons (AAAS, 1998; Clements & Battista, 2000; NCTM, 2000). First, they assumed that the technology alone would improve student learning, ignoring how it might have actually produced affective and cognitive results. Second, because the hardware was assumed to make the difference (as opposed to the teaching or the quality of its software), the new hardware tended to be introduced into classrooms hurriedly on a wave of enthusiasm and public support, but with little time and few resources that were devoted to training teachers to integrate the hardware into their curriculum.

Thirdly, because technology was often hurriedly introduced, its role and purpose in instruction was usually left undefined. Consequently, the producers and marketers of hardware in educational technology could not solve these severe problems without drastic changes in the ways schools selected and implemented hardware.

Effects of Technology on Students

Researchers also found that technologies, media, and materials were not productive when the curriculum content and pedagogy that was implemented through them had no promise or value (AAAS, 1998). Some of the technologies in schools were introduced without attention to how they actually affected student performance. For example, computer-assisted instruction (CAI) was and still is one of the more popular forms of technology used in schools (See Figure 3). However, most CAI programs are highly individualized, a method that was criticized by Hativa (1988) for increasing the gap between high- and low-achieving students. It raised the question of the effectiveness of initial instructional approaches (such as highly individualized instruction) before computer-assisted instruction was used. AAAS (1998) also raised Hativa's (1988) point as a critical one because of the high cost and maintenance of CAI.

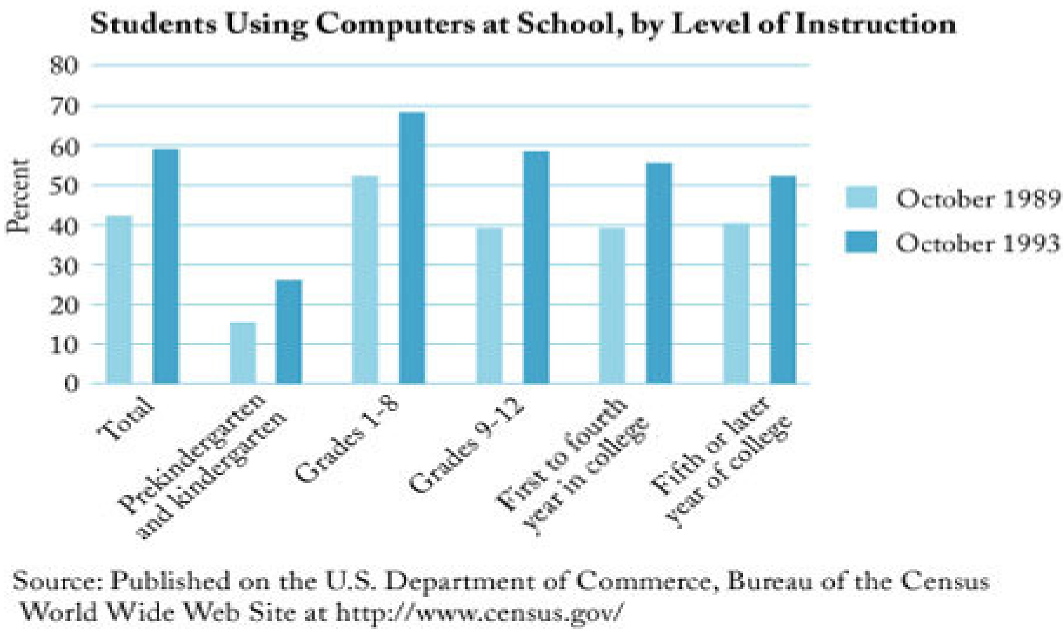


Figure 3. Graph of student computer usage by grade level (from Hativa, 1998).

Cost-effectiveness was always important when considering technology for use in the classroom. For example, if a $1,000-per-student, computer-based program intended to raise achievement did only as well as an existing or cheaper program, then the technology failed in terms of cost-effectiveness. On the other hand, when technologies were selected primarily for their ability to save money, student learning might suffer (AAAS, 1998).

There were studies of uses of technology that did not produce poor results. For instance, researchers discovered substantial advantages of technology over comparison programs, and utilized technology-based programs in ways that delivered effective results (at one-tenth the cost of previous programs) for wide ranges of student ability and content understanding that transferred to new situations (Thorkidsen & Lowry, 1990; Woodward, 1994). And although technology was important for providing access, many researchers attributed positive results to the specific combination of pedagogy and curriculum organization in the program content (Caverly & MacDonald, 2003; MacDonald, Vasquez, & Caverly, 2002; MacDonald, 2001; Lesh, Hoover, & Kelly, 1992; Minstrell, 2002).

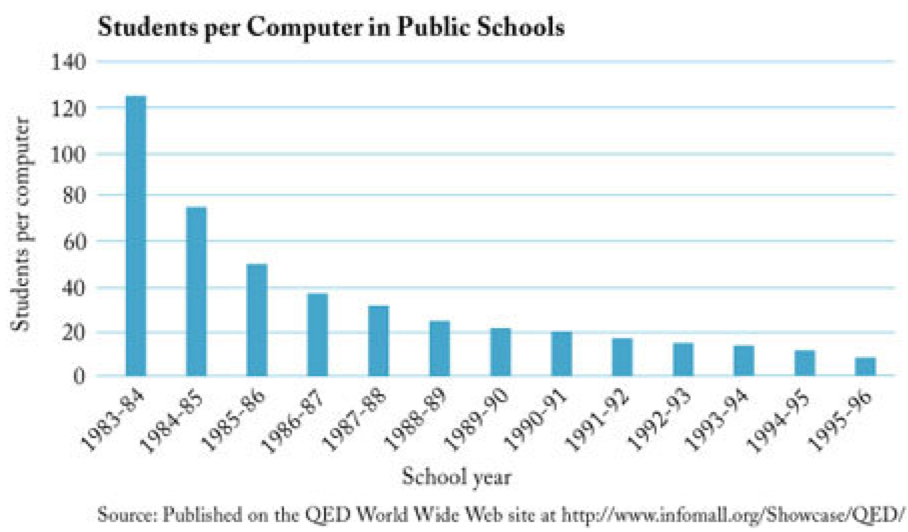


Figure 4. Graph of students per computer in public schools (from AAAS, 1998).

Professional Development

According to some researchers (AAAS, 1998; Ralph & Dwyer, 1988), many educators believed that the major problem when they were disseminating technology-based instructional programs and materials was one of equitable access for all students (See Figure 4). It was argued that many schools, especially in disadvantaged areas, were not designed or equipped for technology-based instruction (See Figure 5). But, even when technology was available, the program could not work unless sufficient training was provided to the teachers who used the technology with students.

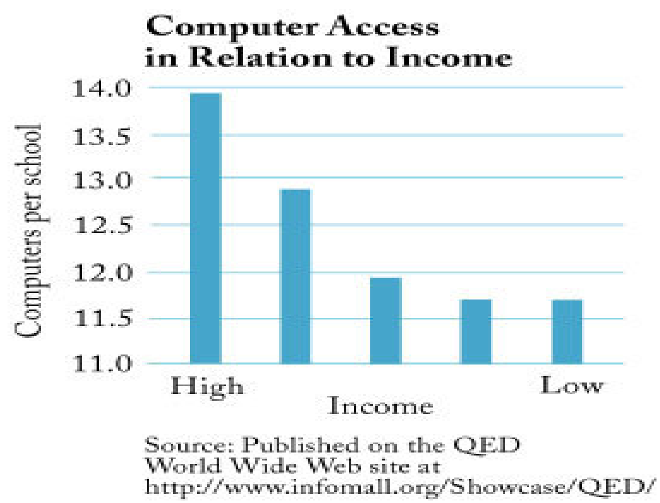


Figure 5. Graph of national student computer access in relation to income (from AAAS, 1998).

Also, in many of the technological reforms, the dissemination of technology-based materials had not been accompanied by staff development that was substantive, program specific, or sustained long enough to be effective. Therefore, the teachers were put in difficult positions. They were often charged with designing instructional materials to accompany technologies that they were not familiar with and where the educational purpose was not properly defined. When the staff development did take place, methods for teaching with a new technology was often prescribed by individuals far removed from the classroom, having little relevance for the unique needs of each teacher's classroom. Therefore, the result of poor staff development, loosely defined goals, and traditional methods of implementation were more of a hindrance in the classroom than help (AAAS, 1998). Callister and Dunne (1992) cited these types of hindrances and lack of teacher preparation (See Figure 6) mainly for loss of teacher control, understanding, and autonomy in the technological environment.

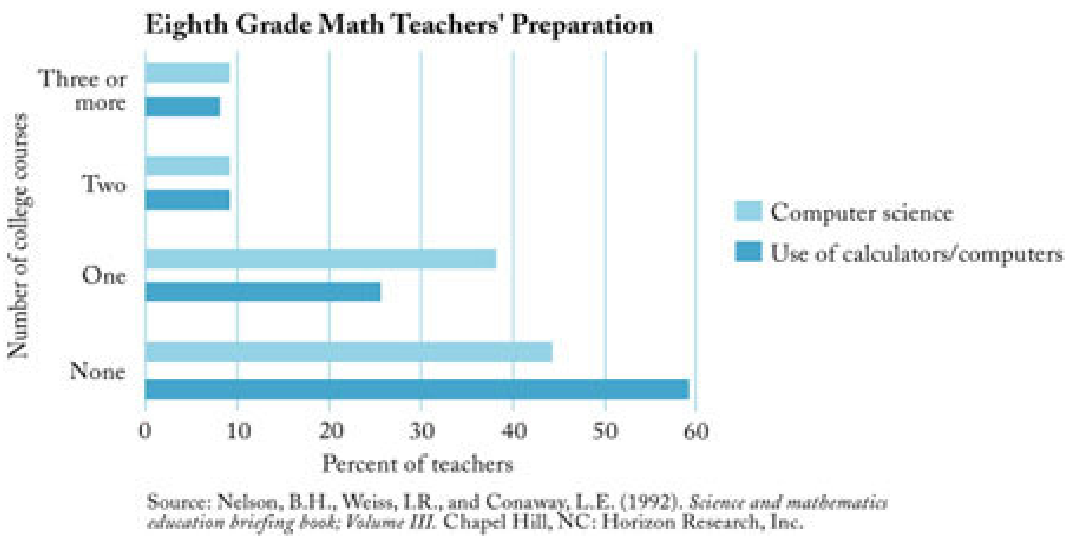


Figure 6. Graph of number of college hours in technology that a sample of teachers had taken (from AAAS, 1998).

Many researchers agree that any effort to send instructional technology and materials down the information highways of the future must recognize that teachers are the most important link in the chain whenever trying to connect technological innovation with improved student performance (AAAS, 1998; Clements & Battista, 2000; MacDonald, 2001; Ralph & Dwyer, 1988).

Use of Technology in the Classroom

Whenever a new technology is introduced into classrooms, its role should be clearly defined before it is used (AAAS, 1998; Callister & Dunne, 1992; Clements & Battista, 2000; Woodward, 1994). It was found that students, teachers, administrators, and marketers of the product alike often misunderstood the role of a certain technology. And when few educational administrators would work with teachers and others to clarify how technology would be used in the classroom, and few marketers of technological hardware worked to determine the unique needs of specific districts or schools in which their products were to be sold, the result was often disastrous (AAAS, 1998).

It was found that failure to clearly define technology's role had several adverse consequences (AAAS, 1998). When a technology-based program was used as the entire curriculum, a certain set of evaluation criteria had to be present to assess its worth. But if this same program was used as a tool to support the learning of some other curriculum goal, an entirely different set of evaluation criteria apply. For example, if an Excel™ spreadsheet program was used, an important question to ask was, "Was it the focus of a curriculum unit or an entire course in which the goal is to teach students how to use an Excel™ spreadsheet program?" When this same program was used in a business class because the teacher believed that using Excel™ spreadsheets improves the quality of students' data collection, analysis, and reporting in quantitative analysis, it could have two purposes. Learning about the spreadsheet as a tool and using the spreadsheet to analyze data within a spreadsheet program has vastly different roles. When those roles were not clearly defined, it was impossible to determine how effective technological intervention was (AAAS, 1998; Woodward, 1994).

Facet Theory and Its Use in Assessment

The original definition of *facets* was broad, allowing for many various usages. Initially, it was Guttman (1958) who originally defined *facets*, saying, “A classification of the item-domains of a given content universe according to some rule is called a facet of that universe” (Shye, 1978, p. 9), clearly allowing for a broad use of the term *facets* in several fields. For example, Shye (1978), Minstrell (2002), and Schumaker (1999) have all used *facets* or *facet* analysis in their research areas to study various hypotheses in social science, physics, and statistics. It is because *facet*-based research allows for the collection of a wide range of data both prospectively and retrospectively that it can be so useful in different studies (Shye, 1978).

Minstrell’s (2001) study of *facets* led to the design of the *Diagnoser,* an online program providing a level of description and procedure for constructing models of student thinking in mostly physics topics that help classroom teachers make instructional decisions. Also, the *Diagnoser* was used to assist researchers by providing empirical data from which to make inferences. The empirical data comes from his theory of *facets* and *facet* clusters. Minstrell

(2002) defines *facet* clusters as “sets of related facets, grouped around explaining or interpreting a physical situation (e.g. Forces on Interacting Objects) or around some conceptual idea (e.g. Meaning of Average Velocity.)” (p.4). But, the *facets* are “used to describe students' thinking as it is seen or heard in the classroom or other learning situation. Also, *facets* of students' thinking can be seen as individual pieces, or constructions of a few pieces, of knowledge and/or strategies of reasoning” (Minstrell, 2002, p.4). To summarize, Minstrell’s (2000) *facet* clusters are the organized *facets* of student thoughts concerning a specific content area (see Table 2).

Table 2. Minstrell’s (2002) Facet Cluster for Explaining Falling Bodies

*340 Gravitational pull by earth on falling object and mass of object compensate for each other. The resistance by the medium through which the object is falling increases with speed and will decrease the rate of acceleration
*341 (Fg – Fr)/ mass = acceleration (instantaneous rate) of fall. With no resistance, near the earth, things fall, accelerating at about 9.8 m/s/s.
342 Gravitational pull and mass compensate, but greater air resistance on the lighter object, making it fall behind.
343 Gravitational pull and mass compensate with no accounting for air resistance.
344 Greater drag effects compensate for greater gravitational pull explaining equal accelerations. No apparent accounting for inertial mass of the object.
345 Drag effects of medium will exist even when there is no motion relative to fluid medium. The resistive force exists even when the object is not yet falling.
346 All things fall with equal acceleration of about 10m/s/s.
347 All things fall equally fast regardless of medium effects. For example, vertical fall is at a constant velocity of 10 m/sec
348 Weight makes it hard to move things. The more weight, the slower they fall. It takes time to get them going. Heavier things will lag behind until they can get going.
349 Weight makes things fall. The more weight, the faster they fall.
349+ When you let things go, they fall
349++ Things fall down.

Note: For the cluster “Explaining Falling Bodies,” the facet ending in 0 again represents a more conceptual understanding and the facet ending in 1 represents the more mathematical modeling of the situation. (Minstrell, 2002, p.22).

Not excluding Minstrell (2001), Borg and Shye (1995) also provide an appropriate definition that closely matches the definition used in this research model. They define *facets* as “a set of elements (i.e., types, classes, categories, attributes, etc.) that classify objects of interest” (Borg & Shye, 1995, p.25). Based on these concepts, and specifically for the purpose of this study, *facets* are defined as the idea units (observed thoughts of students while working on problems in MUL) classifying the known content universe of Multiple Meanings and Models of Fractions (MUL). Also, a *facet* cluster is defined as an arranged set of those idea units.

Diagnoser and Minstrell’s Facets of Thinking

Minstrell’s research stemmed from his “need for a practical language to describe students’ thinking” (Minstrell, 2002, p.4). He expressed in his studies that understanding complexity in the physical world necessitated what, in a 1999 article, Goldenfeld and Kadanoff called “focus on the right level of description… Use the right level of description to catch the phenomena of interest. Don’t model bulldozers with quarks” (Minstrell, 2002, p.11).

In addition, Minstrell’s research found that to model the complexity of teaching and learning in the classroom, you also need to use the right level of description for that purpose. The description of students’ thinking needed to be understood by teachers, by scientists and by researchers on learning; then it should lead to a description level that would serve classroom teachers as they made instructional decisions. Minstrell's focus is on teachers as the primary target consumers. His studies further discussed and reviewed what was found from alternative levels of description and then offered a level of description with which teachers could work – the "Facets of Thinking" (Minstrell, 2002, 2001, 2000). Minstrell (2002) also described how educators could use *facets* in designing science assessment and instruction, and offered guidelines for research and development of *facets* and *facet*-based learning environments.

Minstrell's (2002, 2001, 1989) studies focused on the primary question, if there exist results from research on students' misconceptions, how should the results be organized for effective teacher use? He suggested that some organizations involve characterizing students' thinking as theoretical in the large scale sense of organizing lots of phenomena. For example, were the students' ideas more consistent with Newtonian theory or Impetus theory of motion (Minstrell, 2002)? Other classifications involved identifying tiny phenomenological primitives that come from the perception of features of particular situations. Such as, from the "knowledge in pieces" perspective, students looked for whether the objects involved in a particular situation were perceived to be rigid or springy. Then, there were characterizations that looked at logic of ontological categories of conceptions used by the learners. For example, do students think of force as an "action" on the object or as a "property" of the object (Minstrell, 2001)?

Minstrell's investigation of each of the aforementioned research perspectives resulted in subsequent belief in their validity. At some level, each research perspective described its supporter's beliefs about the nature of student thinking, and each suggested a particular view of a learner's knowledge. But Minstrell's (2000) focus remained with descriptions that could inform issues and decisions concerning curriculum and instruction.

DiSessa's (1993) work provided much of the theoretical foundation concerning the importance of attending to the features of problematic situations similar to that used by Minstrell (2000). DiSessa (1993) explained how learners chose intuitive mechanisms and elements that seemed relevant and constructed an explanation or description from those intuitions. The "knowledge in pieces" perspective was the construct for thinking about the disassembling of learners' existing understanding and reconstruction of new knowledge built from the intuitive pieces. Chi, Feltovich, and Glaser (1981), in an expert/novice study, suggested that we watch for ways in which learners construct knowledge in categories that prevent them from understanding the logic of science. Also, in an earlier study it was suggested that an overall goal for learning may be in developing a different theoretical perspective (McCloskey, Caramazza, & Green, 1980). It was concluded that this development should involve reconstructions from the intuitions with care in clarity of the logical organization of ideas.

While Minstrell (2000) found that aspects of each perspective has its applications in the classroom, he concluded that each by itself was not seen to be sufficient or practical for day-to-day planning and teaching in the classroom, nor for communicating to teachers the purposes in their day-to-day activities. These were all theories of knowledge organization on the part of learners. The language and issues in the perspectives did not speak directly to how to respond to what learners have just said or done or to identifying what to do next with the students. He and other researchers found that the goals needed to be more immediate and relatable to the content teachers were expected to teach and students were expected to learn (Clements & Battista, 2000; Minstrell, 2002, 2001, 2000, 1989).

Using Minstrell's (2002) Facets and Facet Clusters to Design Assessment

Minstrell (2002) suggested that when designing assessment activities teachers should use the *facets* within a cluster to predict the type of answers students might give. And, if the assessments are embedded within the instruction, teachers could identify the *facets* still representative of students' present ideas, and then design activities that address those particular problematic *facets*. Also, he designed computerized software (*Diagnoser*) that assisted with ongoing assessment (Minstrell, 2002, 2000). The questions he designed came in pairs, the first asking, "What would happen if...?" (Minstrell, 2002, p.13) And although the assessment involved multiple-choice answers, each choice was associated with a particular *facet*. Therefore the system would make a preliminary diagnosis of specific, potential difficulty. The second question would then follow up with "What reasoning best justifies your answer?" (Minstrell, 2002, p.13) Again, each answer from which the students chose was associated with a particular *facet* of thought. Consequently, this would give a secondary diagnosis and feedback fitting the particular *facet* diagnosis and the problem addressed. It was believed that these sorts of tools helped classroom teachers monitor and address students' thinking in time to address problematic thinking (Minstrell, 2001).

In addition to the initial development, revisiting the *facets* also enhanced the design of subsequent assessment. For example, long after students had forgotten the slogans that they may have memorized for tests, strong assessment items were used to incorporate many of the features students intuitively invoked. Specifically, Minstrell described a question that was very difficult for the students whose ideas about interacting objects had not yet changed:

> A bowling ball rolls down an alley and hits a bowling pin. How do the forces that the ball and the pin exert on each other compare? In this case the bowling ball had all the advantages from the students' intuitive notions. It was heavier, made of harder material (stronger), moving when it hits the pin, more active as it moves along toward the passive pin, and certainly creates the greater effect, in the sense that the pin gets knocked way back while the ball simply continues to roll along, apparently with little change (Minstrell, 2002, p.13).

These emphases are the key differences between traditional assessment and what Minstrell (2002, 2001) defines as *facet*-based assessment. In the former, assessment tended to look for affirmation as to whether students were responding correctly or not. He designed the *facet*-based assessment questions to be what students called "tricky," in that they offered seductive situations, trying to find out exactly what learners had not yet comprehended as part of their understanding. In the previously stated example, he did not want to know that students had learned to parrot an "equal and opposite" answer as a result of being trained in "school science." He wanted to see if students had actually changed their thinking about the world around them. Therefore, the "tricky" questions that may have been alongside questions asking about the forces on the pin - “Force by bowling ball on pin is bigger than resistive force of friction by floor on pin, but force by ball on pin equals force by pin on ball” (Minstrell, 2002, p.13) – test for a strong conceptual understanding. Learners needed to understand when the slogans "equal and opposite" and "net unbalanced forward force" apply.

While further illuminating his *facet*-based clusters, Minstrell (2002, 2001, 2000) explained that the relation between the action-reaction idea (*facet* cluster 470), the forces on an accelerating object (*facet* cluster 420), and the forces on objects moving with a constant velocity (*facet* cluster 430), and forces on an at-rest object (*facet* cluster 410) need a natural relationship between multiple clusters. Accordingly, lasting development in one cluster would require building a model for understanding a larger concept, but the construction happened with a focus on one cluster at a time. The *facet*-based assessment and related instruction pushed students to learn what they did not yet understand as well as connecting to what they did seem to understand. Learning was a gradual and ongoing process. The diagnostic assessment gave more specifics as to where the student's understanding seemed to be at a particular time so teachers knew what needed to be addressed in instruction (Minstrell, 2002).

Minstrell (2000) also suggested that results from large-scale, on-demand assessments based on *facets* could inform educators and policy makers on where resources are needed. If students in a particular school were stuck on a particular *facet*, that school could be made aware of the curriculum and instruction used by another school that specifically addressed that *facet* (Minstrell, 2002). Further implications included the suggestion that effective lessons could be shared between schools and teachers to address particular learning difficulties, provided the particular problematic thought could be identified. Conclusively, because typical large scale assessment told only generally where the troubles lie, *Diagnoser* tended to provide information about what specifically needed to be addressed and how (Minstrell, 2002, 2001, 2000, 1989).

Design of Facet Clusters

Minstrell's (2002) *facets* were determined by interviewing learners through paper-and-pencil questions and elicited responses and, also through responses to questions on the web. Questions that were used to elicit *facets* were carefully crafted to engage learners' thinking with respect to identified learning targets. Since the purpose of *facet*-based learning environments was to guide assessment and instruction at the learning site, the teachers of the discipline were involved in reviewing and revising *facets* and in evaluating their utility (Minstrell, 2002).

The following outline summarizes the guiding principles used by Minstrell (2002, 2001) for designing a *facet*-based learning environment:

1. General goals -The understanding and process skills that represent the learning goals for the particular target audience were identified.
2. Specific goals -The specific ideas and events critical to what the learner needed to know and was able to do were identified.
3. Elicitation questions -Questions were designed which engaged the thinking of learners with respect to the learning goals.
4. Tentative *facets* -A sufficient number of responses to questions was collected to determine various approaches to thinking about each of the critical ideas and events. (About twenty in-depth interviews, 40 extended written responses to a hundred short responses were a sample recommended by Minstrell (2002)). Also, it was recommended that facets be repeated by at least 10% of the interviewed before they are included in the facet table.

5. Rank ordering of *facets* - It was suggested to order the *facets* from those most problematic to those more consistent with the learning goals (whenever possible). Minstrell (2001) used intellectual development and instructional efficacy to guide criteria.
6. Diagnostic questions - More specific questions were designed from which students' apparent facets could be identified.
   a. Questioning contexts were related to the critical thinking in the goals.
   b. Questioning contexts were seductive to elicit some problematic *facets*.
   c. Specific answers associated with relevant *facets* were written in multiple-choice format or rubrics for *facet* diagnosis of open response format.
   d. An option was left for learners to respond with an unanticipated answer.
7. *Facet* revision - The tentative *facets* and clusters were revised on the basis of how well they described students' responses and apparent thinking.
8. Question revision - Questions were revised on the basis of how well they elicited students' thinking.
9. Feedback - Feedback was designed that fostered open sharing of learners' ideas and stimulated critical thinking on the part of learners.

10. Prescriptive lessons - Lessons were designed based on research of instruction known to effect movement from particular problematic facets of thinking toward thinking that was more consistent with the learning goals.
11. Test and revise the *facet*-based learning environment t- Test of the effects of the learning system was conducted. Costs against benefits of using the system were re-evaluated. The system was reconstructed to improve benefits and lower costs (Minstrell, 2002, 2001).

Minstrell (2002) further iterated that the identification of *facets* and *facet* clusters was an on-going, iterative research and development process. The extension of the contexts of questions around one cluster increased the complexity of the knowledge and understanding required to appropriately address the question. Also, he received unexpected responses that required him to identify new *facets*. Sometimes he divided the clusters in two, but he refocused things and realized that the tool was initially developed to meet the needs of students that were fairly new to the topic. The priority was to first address students' learning needs and their teachers' needs for tools to foster the learning at that level. Then, a more elaborated cluster could be created for the learners who were going on to become experts in the field.

In addition, Minstrell (2002) encouraged the teachers to keep their eyes and ears open for different *facets* that may need to be addressed. He exemplified this point:

> At one point, while investigating forces and gravity, I suggested to my students, “Suppose there were no friction acting on the wheels of the cart. Suppose there was no air resistance acting on the cart, no air in the room…” And then I heard quiet voices in the corners of the classroom saying, “Why then everything would just float off the table.” This meant I needed a facet cluster dealing with the effects of the surrounding fluid (air) as well as a cluster about the nature of gravity and other actions at a distance. Thus, the process of facet finding will likely never be complete. The more complex the learning we expect, the more practical tools we will need to describe what is happening with respect to students’ thinking (p22).

Scaling Facet Data

The underlying concept behind Guttman’s facet theory is to provide a more valid way to quantify and measure qualitative data. According to Levy (1994), Guttman’s theory was originally designed to seek a facet design for mental abilities and eventually boiled down to seeking a definition for mental abilities. Guttman found that “difficulty with the old saw that ‘intelligence is what an intelligence test measures’ was that it was virtually facetless” (Levy, 1994, p. 513). Conclusively, Guttman’s facet theory both defines the universe and scales it for certain types of data, citing the construction of structural hypotheses rather than with inference from samples (Guttman, 1958; Levy, 1994; Shye, 1978).

In addition to the other theories on *facet* analysis, Schumacker (1999) explains the usage of many-facet Rasch analysis. In his studies he explained the use of a crossed, nested, and mixed design of Rasch analysis to use for comparing *facets*. By using this method Schumacker (1999) offers the ability to create a vertical scale for each *facet*. Conclusively, analysis of the coded data by Guttman (1958), Rasch analysis, factor analysis, or various multivariate methods address the question of validity for the final instrument scale (Schumacker, 1999; Shye, 1978; Stevens, 1996; Thompson, 1984, 2000; Zvulun, 1978)

Summary of Review of Literature

Because of an emphasis on the rational number in school mathematics curricula and the concomitant rapid progression to symbolic computation procedures, a disproportionate amount of pure research on rational numbers was concerned with questions relating to which of several algorithmic procedures would best facilitate student's computation performance. Today, research has included data-based observations concerned with attempts to identify and describe the mental processes employed by students engaged in these tasks and not just with simple comparisons between two instructional procedures (Lesh, Lamon, Gong, & Post, 1992).

Currently, a large majority of research consists of status studies. That is, researchers gather data relating to a student's knowledge of a particular area without regard for concurrent instruction or consideration of the quality or extent of the student's past instructional experiences. Because much of what students know about the more formal aspects of mathematics is influenced by instruction, these status studies, although very useful, are inherently limited in the extent to which students' cognitive structures can be linked directly to instruction and/or specific experiences (Lesh, Cramer, Doerr, Post, & Zawojewski, 2002; Post, Cramer, Behr, Lesh, & Harel, 1993). Furthermore, these studies do not provide any insights into how concepts develop over time under the influence of a well-defined instructional sequence. Such information is critical if research is to provide guidance for the redefinition of school curricula to promote the more effective learning of mathematics by all students.

This research focuses on the development and investigation of technology-based diagnostic instrument designed to assess the development of cognitive structures for rational number thinking within a well-defined, theoretically based instructional program.

CHAPTER III

METHODOLOGY

Introduction

This study followed a Research and Design (R&D) model to investigate the validity and effectiveness of the *Fraction Diagnoser* as a diagnostic tool. This chapter explains the methodologies for (1) the design of the instrument, along with the collection and analysis of data to validate the design, and (2) the implementation of the instrument and the collection of data to determine its impact on teachers. Specifically, the study addressed these research questions:

Design Questions:

1. What is the *Facet* Cluster related to multiple meanings and models of fractions (MUL)?
2. (Between subjects) How well did the *Fraction Diagnoser* identify distinct levels of understanding of MUL concepts and skills for individual students in developmental mathematics?
3. (Within subjects) What kinds of student information did the *Fraction Diagnoser* provide to describe student growth toward mastery in MUL?

Implementation Question:

4. How can teachers use the information provided by *Fraction Diagnoser* to make instructional decisions?

Design of the Instrument

The blueprint for this study during the R&D cycle was the model presented in Borg and Gall (1989). Although other models are used and referenced, they all coincide with the same steps referred to by Borg and Gall (1989) (Borg & Gall, 1996; Clements & Battista, 2000; Minstrell, 2000, 2002). Throughout this study, participants changed, depending upon the step of the R&D cycle. So, to clarify the participants involved (student or teacher), the details of the participants are initially addressed in each R&D cycle step (where necessary). Highlights of the procedures of the steps in the R&D cycle of this study, as related to the steps presented by Borg and Gall, are presented in Table 3 and will be elaborated further in the following sections of this chapter.

Table 3. Borg and Gall R&D Cycle Steps with Information Sources

Borg and Gall R&D steps	Information Source or Study Sub-steps
1. Research and collection of background information	a. Initial Learning Goal b. Rational Number Project c. Project 2061/AAAS d. NCTM Standards e. Texas Higher Education Assessment (THEA) Items f. Informal interviews with developmental mathematics instructors at 3 different colleges and universities

Table 3. Continued

Borg and Gall R&D steps	Information Source or Study Sub-steps
2. Planning the procedure of the study	a. Timeline b. IRB Information
3. Preliminary product development	a. Construct Validity of initial items for learning goal b. Initial evaluation of clarification of test questions
4. Preliminary product test	a. The test of *Fraction Diagnoser* to 134 student participants during summer of 2004
5. Product revision	a. Interview of 48 of the 134 student participants who took the *Fraction Diagnoser* b. Organizing of *facets* c. Inclusion of 2 new items
6. Main field test	a. Testing of 112 new student participants three different times during the course of the Fall 2004 semester
7. Operational product revision	a. Revision of *Fraction Diagnoser* during third trial to include equivalent items and to openly include diagnostic interview items b. Random interview verification that diagnostic *facets* corresponded to what student actually thought or did c. Instrument description
8. Operational field test	a. Testing of 300 new student participants once during the Spring 2005 semester
9. Final Product Revision	a. N/A
10. Dissemination and implementation	a. Interview of Teacher participants concerning the usefulness of *Fraction Diagnoser* in the academic environment

Borg and Gall (1996) recommend that for a dissertation it is best to limit instrument development to a few steps of the R&D cycle because of the substantial resources that are necessary for the endeavor of completing the entire cycle. Therefore, the final product revision (Step 9) was not included in the study, but is included in the outline of the R&D cycle in order to exemplify the proper R&D procedure. The researcher labeled the final interviews as part of Step 10 but it could be concluded that the final interviews are part of Step 8 and hence the R&D cycle would be closed there. But due to the components of the interview process being similar to the Borg and Gall (1996) Step 10, the researcher decided to present this data as part of the final step. Regardless, the steps of the R&D cycle used in this study adequately address the research questions and purpose.

Research and Collection of Background Information – Step 1

Although no step in Borg and Gall (1989) specifically addresses the development of the initial learning goal, this initial step was paramount to the development of *Fraction Diagnoser* (Clements & Battista, 2000, Minstrell, 2001). The development of the learning goal is included in this step of the R&D cycle, and because of the well-documented difficulties students have with learning fractions, this topic became the initial learning objective of the diagnostic tool. Later, due to the broad nature of fractions as a topic, the researcher identified a more specific focus--the subgroup of understandings of the Multiple Meanings and Models of fractions (MUL). The MUL learning objective was adapted from the Project 2061 assessment map for fractions (see upper circle in Figure 7).

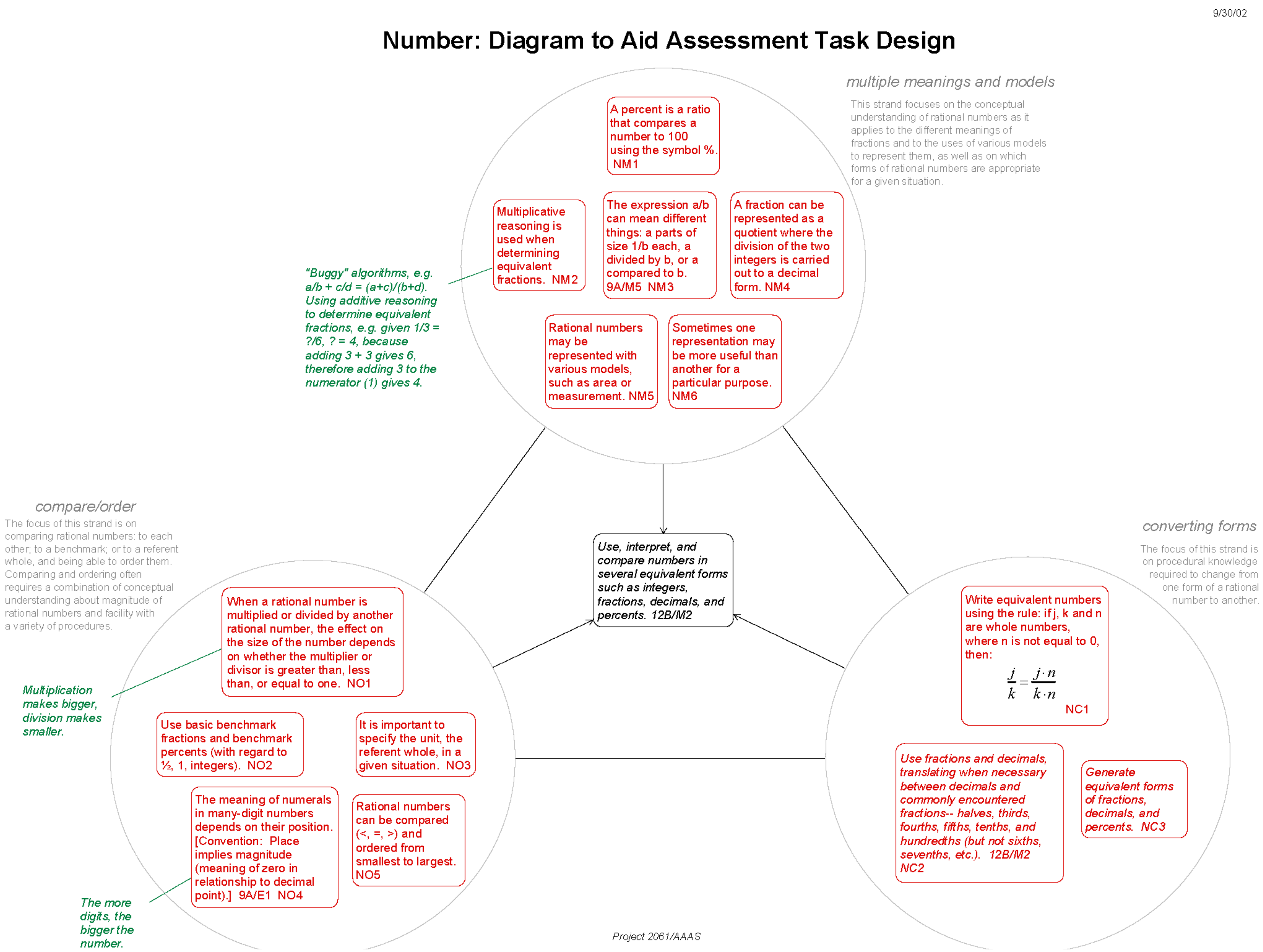


Figure 7. The assessment map for the understanding of fractions developed by Project 2061/AAAS.

Research done by the Rational Number Project was thoroughly reviewed, specifically studies relating to student thinking on the topic of MUL (Behr, Harel, Post, & Lesh, 1994; Behr, Lesh, Post, & Silver, 1983; Bright, Behr, Post, & Wachsmuth, 1988; Cramer, 2001; Cramer, Behr, Post, & Lesh, 1997a, 1997b; Cramer & Post, 1995; Cramer, Post, & Behr, 1989; Cramer, Post, & Currier, 1993; Lesh, Lamon, Gong, & Post, 1992; Post, Cramer, Behr, Lesh, & Harel, 1993). Also, the maps from the Atlas for Science Literacy (AAAS, 2003) and Project 2061 (see Figure 3) were evaluated and compared to ensure that the content domain for the chosen objective was covered completely. Along with the maps, the content standards published by the National Council of Teachers of Mathematics (NCTM, 2000) along with the Texas Essential Knowledge and Skills (TEKS) were compared to clearly define the skill domain for MUL. The assessment map for subgroup multiple meanings and models from the AAAS map of Number was used as the foundation skills for MUL (see Figure 8).

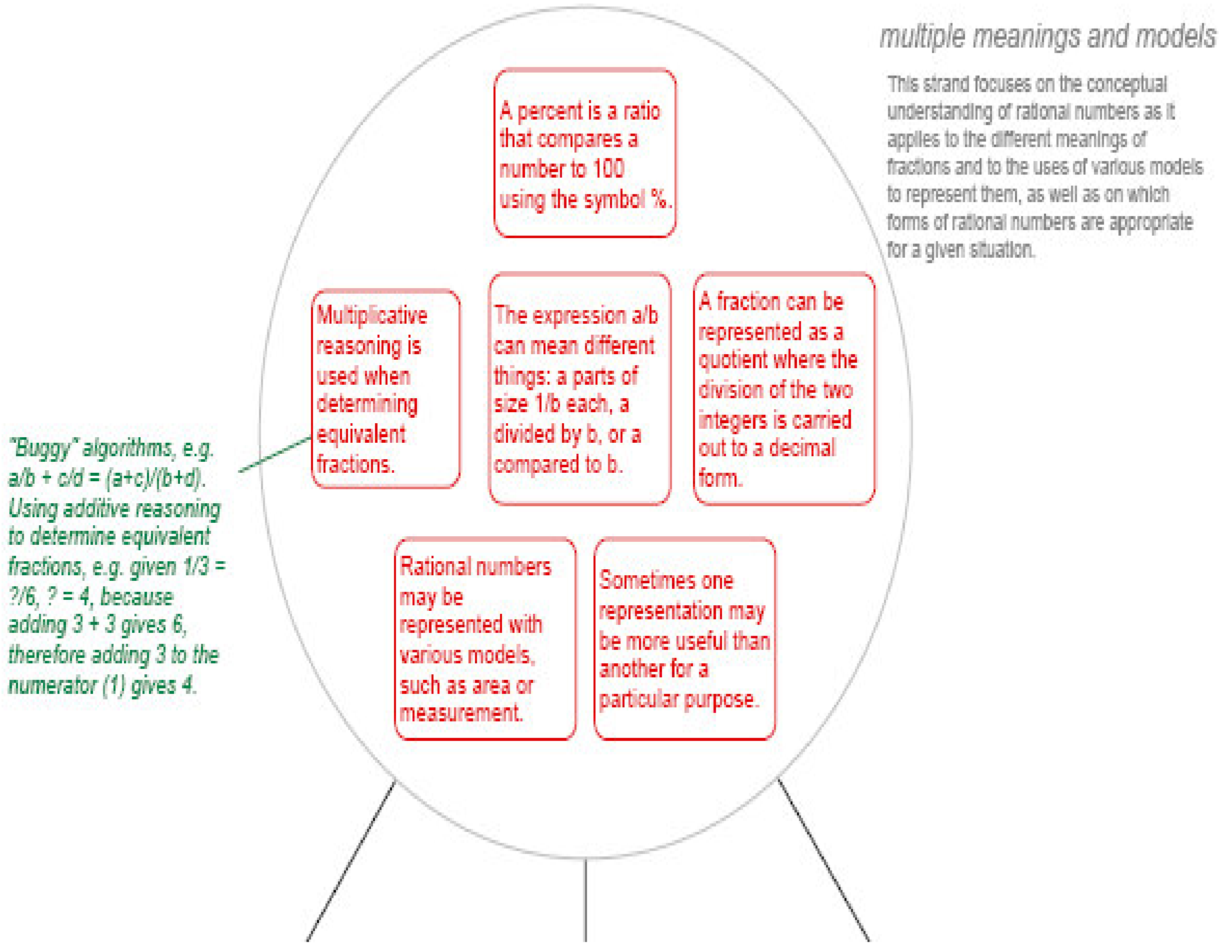


Figure 8. The foundation skills for domain multiple meanings and models of fractions.

In addition to searches done for the content learning objective (MUL), the researcher gathered information from experts in the field of teaching mathematics to developmental students concerning the student population. Informal interviews were conducted with instructors from four different universities and colleges in east central Texas to gather information concerning the various programs handling developmental mathematics students. This was done because the state allows a certain amount of freedom to the university concerning programs for developmental students, and the placement and instruction for these students are handled differently at different institutions. Curricula and instruction for classes based on levels of achievement on the THEA were not consistent across institutions, including some cases where independent study was allowed. Therefore, for the scope of this study the population was limited to the developmental students who received class instruction for pre-algebra courses.

Planning the Procedure of the Study – Step 2

The chronology for this study is shown in this timeline (see Figure 9).

Nov-03	Dec-03	Jan-04	Feb-04	Mar-04	Apr-04	May-04	Jun-04	Jul-04
Step 1								
		Step 2						
			Step 3					
							Step 4	
								Step 5
Aug-04	Sep-04	Oct-04	Nov-04	Dec-04	Jan-05	Feb-05	Mar-05	Apr-05
Step 5								
Step 6				Step 7	Step 8			
						Step 10		

R & D Cycle

1. Research and collection of background information
2. Planning the procedure of the study
3. Preliminary product development
4. Preliminary product test
5. Product revision
6. Main field test
7. Operational product revision
8. Operational field test
9. Final Product Revision (Excluded from study)
10. Dissemination and implementation

Figure 9. Timeline for study: This gives a pictorial timeline to show reference to study plan.

The following summaries are explanations, referencing each research question, of when particular steps of the R&D cycle were the focus of the study during the timeline (a more detailed description in each particular subsection written for each step of the R&D cycle):

Validation Questions:

1. What is the *Facet* Cluster related to multiple meanings and models of fractions (MUL)?

For this research question, Steps 1, 3, 4, and 5 were most pertinent, showing that the *Facet* Cluster was revised throughout the study. This evolution of the cluster is not an uncommon situation in *facet* research (Minstrell, 2002, 2001, 2000, 1989). It was hypothesized that the final *facet* cluster for MUL would yield a more detailed description of student thought.

2. (Between subjects) How well did the *Fraction Diagnoser* identify distinct levels of understanding of MUL concepts and skills for individual students in developmental mathematics?

Steps 6 and 8 of the R&D addressed this research question. Reliability and validity measures were employed to determine how accurate the instrument would be. During these steps one group of student participants were tested three times during the course of the semester. These student participants were tested once before instruction, then again after instruction, and lastly a month later. A test-retest reliability coefficient was estimated using data from the first two trials. After the first two trials and the final revision to *Fraction Diagnoser*, student scores were analyzed using descriptive and inferential statistics.

Specifically, the data in the last administration of the instrument in Step 6 was also used with Step 8 data to address both convergent and discriminant construct validity. *Fraction Diagnoser* was constructed with items that should be correlated, specifically, the Assessment and Diagnostic items. Also, due to the differences in content, the correlations between the *Fraction Diagnoser* and the THEA Test Number Subgroup should not be statistically significant.

Also, a regression analysis was done to address predictive value of the *Fraction Diagnoser* (Borg & Shye, 1995; Huck, 2000; Schumaker, 1999; Shye, 1978; Stevens, 1996). The regression analysis was done using *Fraction Diagnoser's* different item scores as predictor variables and the student participants' scores on the standardized THEA test number subgroup as the criterion variable. It was hypothesized that the *Fraction Diagnoser* items could be used to predict the THEA number subgroup.

3. (Within subjects) What kinds of student information did the *Fraction Diagnoser* provide to describe student growth toward mastery in MUL?

For the above research question, descriptive data was collected to show the information that *Fraction Diagnoser* provided. This data was organized during Steps 5 and 7.

Implementation Question:

4. How could teachers use the information provided by *Fraction Diagnoser* to make instructional decisions?

Step 10 of the R&D cycle addressed the above research question. Interview data concerning the effectiveness, efficiency, and usefulness of the instrument was collected from each teacher participant and other methods were used for triangulation (Griffee, 2005). It was hypothesized that the data given by the *Fraction Diagnoser* could be used to make instruction decisions in a college developmental mathematics classroom.

Preliminary Product Development – Step 3

During this step in the cycle, work was done in order to construct the initial items for the instrument and address early issues of validity. In order to determine which research-based objectives for MUL for developmental mathematics students would be used, the TEKS, AAAS/Project 2061, Research Number Project, and NCTM standards were compared. After the learning objective had been clearly defined, unpacking the MUL learning goal led to a detailed checklist of skills for item development (see Appendix C). Then, research was done to find similar items that tested each skill listed. Equivalent sets of assessment items were then created and reviewed by content experts in developmental mathematics. These same experts in developmental mathematics were also used to assure assessment item appropriateness. The Kulm (2004) paper was consistently used and referenced as a guideline to create these assessment items and to assure their suitability for the study. Therefore, both the objectives and the test items were reviewed to ensure that they covered the skills required to master MUL. The skills checklist provided the scoring criterion for the assessment items. In addition, an *a priori* MUL specific *facet* cluster, developed from experienced experts, and theoretically based from Minstrell's design for falling bodies (see Table 2, p. XX), was developed and used to organize and assess student *facets* of understanding for multiple representations of fractions (see Chapter 2 for Minstrell's steps for designing *facet*-based assessment).

Also, number items from the THEA mathematics practice test were reviewed (see Appendix A). Because the THEA test Number subgroup score is used as the criterion variable in the validation of the instrument, the THEA items related to number also served as a reference as to what skills would be required of student participants. The skills for the THEA number subgroup were defined as follows: "Solve word problems involving integers, fractions, decimals, and units of measurement. Includes solving word problems involving integers, fractions, decimals (including percents), ratios and proportions, and units of measurement and conversions (including scientific notation)." (http://www.thea.nesinc.com/). Therefore, a simple item analysis, referencing Kulm (2004) and Borg and Gall (1996), was done by a group of experts to verify the MUL skills that were used on content-related THEA items.

Instrument (Fraction Diagnoser at Step 3)

Fraction Diagnoser in this step of the R&D cycle consisted of nine assessment items. These items were designed by a group of experts to cover the skills used to understand the MUL objective. They were all multiple choice items with four possible responses (see Appendix B).

Preliminary Product Test – Step 4

Participants

Approximately 130 pre-college students assigned to a university in southeast central Texas participated in the preliminary product test of *Fraction Diagnoser*. Although these student participants were not developmental mathematics students enrolled at the university, these student participants were in a summer program targeting potential developmental students. Also, the program gave pre-algebra instruction to the students identical to the developmental mathematics curriculum of the university participating in the later part of the study. Therefore, the student participants were a viable sample representative of the study's population.

Procedure

The student participants were given the initial *Fraction Diagnoser* items (see Appendix B). These items included only the assessment items for the *Fraction Diagnoser*, not the diagnostic questions, which were to come later after the interviews in Step 5 of the R&D process. The initial *Fraction Diagnoser* items were given in paper form with the purpose to gain *p* values for items, reevaluate item validity, and give elicitation questions designed to attain student *facets* of thought (Kulm, 2004; Minstrell, 2002). The procedures followed coincided with Minstrell's (2002) guiding principles for designing a *facet*-based environment. After the student participants were given the items, 48 of the students were randomly chosen to be interviewed to find their particular *facets* of thought for each item. Each student's *facets* for the initial nine items were collected, and then organized to develop a revised *facet* cluster to explain students' various ways of thinking about MUL.

Product Revision – Step 5

The product revision step of the study consisted of the collection and organization of the *facets*. Table 4 provides the questions asked of the student participants in order to collect data concerning their *facets* of thought for each particular question.

Table 4. Interview Guide for *Fraction Diagnoser*: Collection of *Facets* for MUL.

(Start by self introduction and explanation of interview to gain casual association.) This is a review of your math diagnostic assessment given _________. I would like to understand what your thought process was at the time you were taking this diagnostic, and there is no particular correct way of responding to any of my questions. The goal is to understand what strategies or methods you may have used and why. Your information will assist in the design of an online instrument that will focus on diagnosing these strategies. Do you have any questions for me? Then I have a few specific questions I would like to ask you.

1. What strategy did you use to come to this answer?
   Why did you use that strategy?
2. What strategy did you use to come to this answer?
   Why did you use that strategy?
3. What strategy did you use to come to this answer?
   Why did you use that strategy?
4. What strategy did you use to come to this answer?
   Why did you use that strategy?
5. What strategy did you use to come to this answer?
   Why did you use that strategy?
6. What strategy did you use to come to this answer?
7. What strategy did you use to come to this answer?
   Why did you use that strategy?
8. What strategy did you use to come to this answer?
   Why did you use that strategy?
   - In cases where a student's strategies 6-8 were confusing ask the following: Which is larger .03 or .003?
9. What strategy did you use to come to this answer?
   Why did you use that strategy?
   - In cases where student responses were inconsistent, ask the following: Where is 2 ½ on the number line (Draw or show a number line to include 2 ½, for example, from -5 to 5)?

The above guide was modified from the California Mathematics Project guide for interviewing teachers who had worked with teacher leaders (Borg & Gall, 1996). After the student participants were interviewed, the organization of the *facet* data began. This organization consisted of the alignment of each question, then the association of each response to relative similarity. Afterwards, the similar responses were used to create the *facets* for each particular question and then to rank order the *facets*. To rank the *facets* from least to most problematic responses, the content experts used both research data of student thought in MUL and the California Mathematics Council's Rubric for Open-Ended Questions as references or scoring guides (Behr, Harel, Post, & Lesh, 1994; Behr, Lesh, Post, & Silver, 1983; Bright, Behr, Post, & Wachsmuth, 1988; Cramer, Henry, 2002; Cramer, & Post ,1995; Cramer, & Post, 1993a; Cramer, & Post, 1993b; Cramer, Post, & Behr, 1989; Cramer, Post, & Currier, 1993; Kulm, 1994; Lesh, Lamon, Gong, & Post, 1992; Post, Cramer, Behr, Lesh, & Harel, 1993).

The final revision consisted of the inclusion of two new items. These items were subsidiary items that came from the interviews and were used to further confirm *facets*. After the conclusion of the interviews, the researcher deemed it necessary to include these questions in order to help correlate the *facets*.

Instrument (Fraction Diagnoser at Step 5)

The *Fraction Diagnoser* evolved in this step of the R&D cycle to include two additional assessment items. During this stage of the development it was found that these subsidiary questions may add more clarity as to the student's thought process and skill ability. These questions were derived from the interview and were not multiple-choice, but identification questions that helped to assess a deeper understanding of items 8 and 9. In conclusion the *Fraction Diagnoser* at this step became an online test of 11 assessment items covering the objective MUL.

Main Field Test – Step 6

Participants

The main field test included 96 student participants in the first trial and 87 in the second. The student participants were developmental (pre-algebra) level mathematics students at a community college and a university in southeast central Texas. Students are placed in developmental courses in the state of Texas if they fail to meet the minimum standard of 230 on the mathematics section of the Texas Higher Education Assessment (THEA) test (National Evaluation Systems, 2003). The students who participated in the study reflected a variety of cultural backgrounds, economic levels, and key demographic factors.

Procedure

The *Fraction Diagnoser* was administered to the students three times during the course of the semester. At the beginning of the semester the student participants took the original instrument with no feedback. Then, after a month of instruction that included no direct instruction of MUL subject matter, the same instrument was administered to the student participants. This procedure was necessary in order to address reliability. This test-retest method yields the reliability coefficient used for the instrument throughout the study, along with comparison to the split-half reliability measure. Finally, at the end of the semester, during the third administration of the *Fraction Diagnoser*, a third version of the *Fraction Diagnoser,* with 11 new items judged equivalent to the first by a panel of experts, plus additional diagnostic questions, was administered on line. Also, the student participants were routinely interviewed to verify clarity of items (both diagnostic and assessment) and *facets*.

It is necessary to state that the *Fraction Diagnoser* for this step in the R&D cycle was online and fully operational. The questions were recreated and transferred using a test building computer program and hosted at www.boocent.com (see Figure 10).

This picture actually depicts what will be seen when a user comes to boocent.com (See Figure 10). The actual assessment is masked within the website and only with proper instructions can a casual observer find the portal. Consequently, each instructor had to be given instructions as to where to find the portal to the assessment and the various data that is collected by the instrument once the process has started.

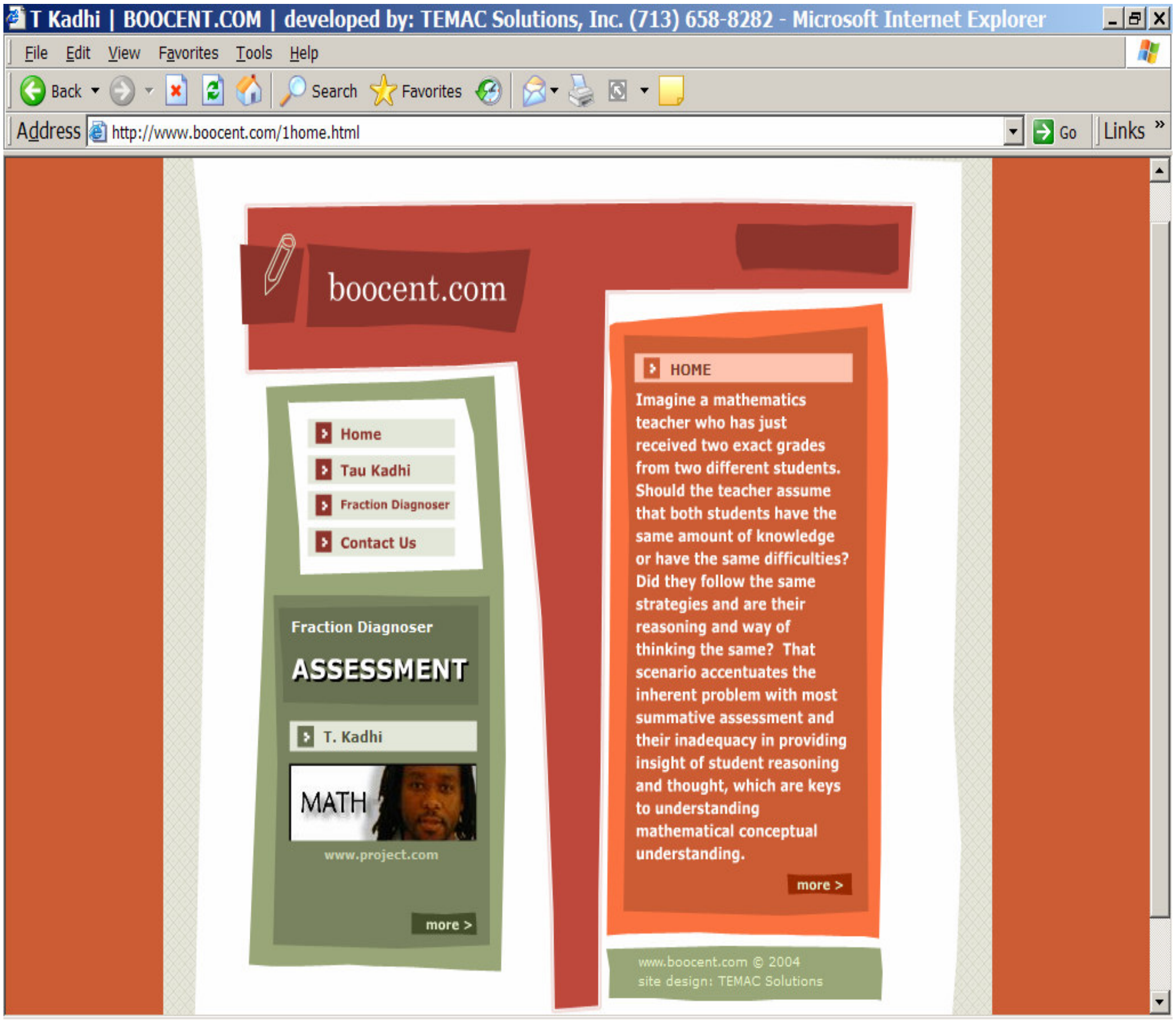


Figure 10. A picture of the webpage that hosted Fraction Diagnoser.

There were a variety of tools that could have been used to create and transfer *Fraction Diagnoser*, but the windows based test-building program Web Quiz XP was eventually used (see http://www.smartlite.it/en2/products/webquiz/index.asp).

Operational Product Revision – Step 7

For the third and final trial in the fall semester, the decision was made to include the diagnostic questions in order to further address the first research question, "What is the *Facet* Cluster related to multiple meanings and models of fractions (MUL)?" After the *facets* were collected for each item, a correlation of *facets* could be made by creating diagnostic questions that asked students to identify a particular *facet* (Minstrell, 2002). This reasoning explicitly follows Minstrell's (2002) guidelines. Also, informal interviews were conducted with randomly selected student participants at the end of the semester in order to verify the clarity of the items and the added diagnostic questions.

Instrument (Fraction Diagnoser at Step 7)

In its third form, the *Fraction Diagnoser* consisted of 11 assessment items and 7 diagnostic questions. The assessment items represented the skills required to understand the objective MUL, while the diagnostic questions documented and assessed the student *facets*. Because MInstrell (2002) described the *facet* cluster as a rough ordinal scale of student *facets*, this study sought to validate the use of such an instrument in developmental mathematics to assess both skills and *facets*. *Fraction Diagnoser* was designed to provide both summative and formative assessment data. The summative data was organized using both a checklist sheet that assigned points to each assessment item based on skills required to complete and a rubric for holistic scoring of the diagnostic items or *facets* based on the California Mathematics Council's open-ended questions rubric (Kulm, 1996).

The Implementation of Fraction Diagnoser and the Collection of Data

Operational Field Test – Step 8

Participants

Seven teacher participants and 334 of their students assigned in developmental mathematics courses at a university in southeast central Texas participated in the Operational Field Test in order to respond to the research question, "How could teachers use the information provided by *Fraction Diagnoser* to make instructional decisions?" These teacher participants represent all of the instructors teaching Math 0100, 0200, 0300, which are all developmental mathematics courses at that university. These teacher participants spend three contact hours per week with the student participants in a lecture classroom environment. These teacher participants are experts in teaching developmental mathematics with a range of 4 to 29 and an average of 10.86 years of teaching experience. The teacher participants volunteered to work in the development of *Fraction Diagnoser* and to administer the instrument as an assignment in their classes.

Procedures

Before the start of the semester each teacher participant received an individual 20-30 minute training session on the use of the *Fraction Diagnoser* and its benefits. The training session taught the teachers how to show the student participants how to log to on to the site and start *Fraction Diagnoser*. Each teacher actually took the *Fraction Diagnoser* in this form and gave feedback as to the clarity. Also, the training session was used to remind them to use the *Fraction Diagnoser* only as an assignment and not as volunteer work, as volunteerism may skew data (Borg & Gall, 1996). All teachers displayed confidence in their ability to administer and use the instrument.

Dissemination and Implementation – Step 10

Participants

Seven teacher participants were used in this step of the R&D. These teacher participants were the same teacher participants used in the Operational Field Test step of the cycle. They were all teachers in developmental mathematics courses at a university in southeast central Texas teaching the entire Math 0100, 0200, and 0300 course load. Also, all of these teachers participated in other steps of the R&D, particularly in the design of test items and analysis of *facets*. The group consisted of 5 males and 2 females. The group represents almost 70 years of teaching experience in developmental mathematics.

Procedures

In order to respond to the research question “How could teachers use the information provided by *Fraction Diagnoser* to make instructional decisions?”, teacher participants responded to the questions in the interview guide in Table 5.

Table 5. Interview Guide for *Fraction Diagnoser*: Interview of Teacher Participants

(Start by self introduction and explanation of interview to gain casual association.) This is a review of the math diagnostic assessment (*Fraction Diagnoser*) given during the _________ semester. I would like to ask you a few questions concerning the basic use and effectiveness of the *Fraction Diagnoser*. Specifically, the research question I am addressing with this interview is “How could teachers use the information provided by *Fraction Diagnoser* to make instructional decisions?” Your responses will assist in the design and development of the *Fraction Diagnoser*. Do you have any questions for me? Then I have a few specific questions I would like to ask you:

1. On a scale from 1-10, with 1 representing needing major changes and 10 being needing no improvement, how would you rate the *Fraction Diagnoser* in terms of effective content for your subject area?
2. Would you care to elaborate?
3. On a scale from 1-10, with 1 representing extremely difficult and 10 representing effortless ease, how would you rate the *Fraction Diagnoser* in terms of difficulty of use for the student?
4. Would you care to elaborate?
5. On a scale from 1-10, with 1 representing extremely difficult and 10 representing effortless ease, how would you rate the *Fraction Diagnoser* in terms of difficulty of use for you (teacher participant)?

Table 5. Continued

6. Would you care to elaborate?
7. In your expert opinion, what are the three major areas of difficulty for students in developmental mathematics?
8. On a scale from 1-10, with 1 being completely useless and 10 being extremely useful, how useful do you think the *Fraction Diagnoser* results are?
9. Do you care to elaborate?
10. How could you use the *Fraction Diagnoser* (show results) results information to make instructional decisions?

After responses to the questions were collected, they were transcribed, coded, and analyzed (Borg & Gall, 1996; Griffee, 2005).

Summary of Methodology

In this chapter, the researcher explains the steps taken in both the design or validation of *Fraction Diagnoser* and the implementation of the instrument. The reader must understand that there are two parts to this study. The R&D that explains which methods were taken to develop an instrument that assesses student understanding in MUL while documenting their *facets* of thought, and an implementation of the instrument and the collection of qualitative data to assess instrument effectiveness in the classroom.

Also, the participant focus changes during this study may be confusing. During the design of the R&D cycle the study participants were mainly students, but during the implementation the participants became their teachers. Early in the study, the students were used to find *facets* in MUL, and then later teachers were used to find out how effective *Fraction Diagnoser* could be as classroom assessment.

In addition to the Borg and Gall (1996) step by step R&D cycle explained in this chapter, the researcher also gave explicit explanations as to why different analyses were done and what was expected to find both during the development and the implementation of *Fraction Diagnoser*. In this chapter the focus was to explain how the R&D cycle was used to address each research question.

CHAPTER IV

RESULTS AND FINDINGS

Introduction

Fraction Diagnoser is an online assessment instrument that was developed by the author during 2004-2005. The design of the instrument was based on the ideas and components of the online instrument *Diagnoser* developed by Minstrell (2000, 2002). *Diagnoser* is an online instrument that addresses learning objectives in physics and other natural sciences and documents student *facets* of thought. This study sought to provide a blueprint for the development of this type of instrument in mathematics content areas, specifically in fractions.

Minstrell (2000) used the term *facet* to describe a particular idea unit of student understanding of a particular topic. It is hypothesized that if teachers identify and use the *facets* in Multiple Meanings and Models of Fractions (MUL), it will help them make instructional decisions in the classroom. Therefore, similar to *Diagnoser*, *Fraction Diagnoser* is an instrument that documents and assesses a student's particular *facet* of thought for the content objective (MUL - Multiple Meanings and Models of Fractions). This study focuses on the validation of *Fraction Diagnoser* as a form of diagnosis and assessment for students enrolled in college developmental mathematics courses.

This study addresses the following particular research questions:

Validation Questions:

1. What is the *Facet* Cluster related to multiple meanings and models of fractions (MUL)?
2. (Between subjects) How well did the *Fraction Diagnoser* identify distinct levels of understanding of MUL concepts and skills for individual students in developmental mathematics?
3. (Within subjects) What kinds of student information did the *Fraction Diagnoser* provide to describe student growth toward mastery in MUL?

Implementation Question:

4. How could teachers use the information provided by *Fraction Diagnoser* to make instructional decisions?

This chapter focuses on the research questions investigated during the research and development phases of *Fraction Diagnoser*. Summaries of findings follow each research question, in addition to related details regarding the development of the instrument.

Research Question 1 - Validation: What Is the Facet Cluster Related to Multiple Meanings and Models of Fractions (MUL)?

Evolution of Facet Cluster for MUL (A priori)

To address this research question a clear definition of *facets* was adapted in Step 1 of the R&D. (The R&D steps referred to in this chapter are defined in Chapter 3). The definition, "Idea units that classify a known content universe," was used. In the specific case of *Fraction Diagnoser*, the known content universe of MUL was used. Then, early in Step 3, a group of experts constructed an item to cover some of the skills used to understand MUL. This open-ended item (see figure 11) was presented to 10 participants.

Find the numbers that equal the BLACK area of this figure (decimal, percent, and fraction)?

Figure 11. A visual of the open-ended question used during the informal preliminary interviews for *Fraction Diagnoser*.

After watching the strategies used by these participants, the researcher constructed an *a priori* table (see Table 6).

Table 6: A Priori Facet Cluster of MUL

Facet Code	Cognitive Characteristics
00	Correctly represents equivalence of fractions, decimals, and percents using diagrams and numerals.
01	Correctly represents equivalence of fractions, decimals and percents with diagrams, but numerals for percents are not correct.
02	Correctly represents equivalence of fractions, decimals, and percents with diagrams, but numerals for decimals are not correct.
03	Correctly represents equivalence of fractions, decimals, and percents with diagrams, but numerals for fractions are not correct.
04	Correctly represents equivalence of fractions, decimals, and percents with diagrams, but numerals for decimals and percents are not correct.
05	Correctly represents equivalence of fractions, decimals, and percents with diagrams, but numerals for fractions and percents are not correct.
06	Correctly represents equivalence of fractions, decimals, and percents with diagrams, but numerals for fractions and decimals are not correct.
07	Correctly represents equivalence of fractions, decimals, and percents with diagrams, but numerals for fractions, decimals and percents are not correct.
08	Correctly represents equivalence of fractions, decimals, and percents with numerals, but diagrams for percents are not correct.
09	Correctly represents equivalence of fractions, decimals, and percents with numerals, but diagrams for decimals are not correct.

After the table was constructed, a group of experts met again to create more items for the *Fraction Diagnoser* (Step 3 of R&D) to cover skills of MUL. A review of the content domains of MUL led the group to outline the following general skills checklist for the items:

1. Student can compare decimals, fractions, and percents.
2. Student can show multiplicative reasoning when looking for equivalency.

3. Student can determine that in the fraction *a/b*, 1/*b* represents equal sections.
4. Student can convert *a/b* to a decimal by dividing *a* by *b*.
5. Student can see rational numbers in models of area or measurement.
6. Student can see a model being more useful than another, depending on purpose.

Items for MUL were then constructed to cover the entire skills domain. Eventually, nine items (see Appendix B) were created in Step 3 for the first iteration of *Fraction Diagnoser*.

Evolution of Facet Cluster for MUL (Facet-based)

After the items were created in Step 3 of the R&D cycle, they were tested in Step 4 in which 48 student participants were interviewed in Step 5 to collect the student *facets* of thought. This collection of *facets* followed Minstrell's (2002) guidelines for creating a *facet*-based learning environment, which are as follows (see Chapter 2 for more specific explanations):

1. Find general goals.
2. Then find specific goals.
3. Create elicitation questions.
4. Identify tentative *facets*.
5. Rank order *facets*.
6. Create diagnostic questions.

7. Revise *facets* based on how well they describe student responses.

8. Revise questions to elicit better student thought.

After reaching the seventh guideline (*facet* revision), the researcher created a table different from the *a priori* one (see Table 7):

Table 7. *Facet* Cluster for MUL

Facet Ranks	*Facet* Codes	
Facets 0	01	I counted each black equal part and used a calculator to convert the fraction to other numbers.
	02	I counted the black part and made it a fraction of the whole.
	03	I reduced the number given and saw the equivalent picture.
	04	I used a calculator to make fractions into decimals and compared.
	05	I found the common denominator on all of the fractions and compared.
Facets 1	01	I cut the pie into equal pieces and saw the equivalent.
	02	I thought any 3-D figure has volume.
	03	I found the fraction for that arrow and saw they were different.
Facets 2	01	I calculated each piece and then added all of the black pieces together.
	02	I looked at the number line and saw that the arrow was less than where it should be.
Facets 3	01	I used process of elimination, because the other answers don't make sense.
Facets 4	01	I connected the two black areas in my mind and made comparisons without using a calculator.

Table 7. Continued

Facet Ranks	Facet Codes
Facets 5	02 I made visual comparisons of the overall black area and just saw what the answer was. 03 I thought they all had volume.
Facets 6	01 I just thought it was common sense. 02 I saw the numbers rising and knew that the numbers were getting bigger (or smaller). 03 I thought that the fraction can be on a number line, but not this one.
Facets 7	01 I counted each black part over the whole and including the hole in the middle. 02 I saw 4 total pieces. 03 I counted the number of pieces and chose the one with the closest number. 04 I saw the arrow at the same number as the numerator.
Facets 8	01 None of these responses reflect what I thought. 02 I thought none of them had volume. 03 I thought circles had volume.
Facets 9	01 I just chose the answer that made the most sense to me. 02 I thought there wasn't enough information to answer the question.

After reviewing the final MUL *Facet* cluster and comparing it to Minstrell's (2000), it was clear that the revising of the *facet* cluster for MUL needed to provide more of a student thought-based table.

Evolution of* Facet *Cluster for MUL (Final)

The facet clusters evolved during Step 7 of the R&D cycle to include a numbering system that was more interrelated with the *Fraction Diagnoser*., resulting in the development of Table 8.

Table 8. Final *Facet* Cluster for MUL

Facet Ranks	*Facet* Codes
Facets 0	020 - I counted each black equal part and used a calculator to convert the fraction to other numbers. 040 - I counted the black part and made it a fraction of the whole. 060 - I reduced the number given and saw the equivalent picture. 080 - I counted each black equal part and used a calculator to convert the fraction to other numbers. 140 - I used a calculator to make fractions into decimals and compared. 140 - I found the common denominator on all of the fractions and compared.
Facets 1	061 - I cut the pie into equal pieces and saw the equivalent. 101 - I thought any 3-D figure has volume. 161 - I found the fraction for that arrow and saw they were different.
Facets 2	082 - I calculated each piece and then added all of the black pieces together. 162 - I looked at the number line and saw that the arrow was less than where it should be.
Facets 3	043 – I used process of elimination, because the other answers don't make sense.
Facets 4	024 – I connected the two black areas in my mind and made comparisons without using a calculator. 084 – I connected the two black areas in my mind and made comparisons without using a calculator.

Table 8. Continued

Facet Ranks	*Facet* Codes
Facets 5	025 - I made visual comparisons of the overall black area and just saw what the answer was. 085 - I made visual comparisons of the overall black area and just saw what the answer was. 105 - I thought they all had volume.
Facets 6	046 - I just thought it was common sense. 146 - I saw the numbers rising and knew that the numbers were getting bigger (or smaller). 166 - I thought that the fraction can be on a number line, but not this one.
Facets 7	027 - I counted each black part over the whole and including the hole in the middle. 067 - I saw 4 total pieces (numerator dependent). 067 - I counted the number of pieces and chose the one with the closest number (denominator dependent). 167 - I saw the arrow at the same number as the numerator.
Facets 8	008 - None of these responses reflect what I thought. 108 - I thought none of them had volume. 108 - I thought circles had volume.
Facets 9	009 - I just chose the answer that made the most sense to me. 169 - I thought there wasn't enough information to answer the question.

Note: First 2 digits of the code for *Facet* Codes represent the number for the appropriate diagnostic question.

Although this cluster became the final cluster for MUL, this table could have been further revised to express smaller clusters more specific to learning goals in MUL. But this research question is addressed by this table expressing all of the MUL *facets* verbalized by these students.

Research Question 2 – Validation: (Between subjects) How Well Did the Fraction Diagnoser Identify Distinct Levels of Understanding of MUL Concepts and Skills for Individual Students in Developmental Mathematics?

To address this research question, reliability and validity were the focus. The data from the study was used to obtain a test-retest and split-half reliability coefficient, to evaluate convergent and discriminant validity, and to find the predictive value of the *Fraction Diagnoser*. Since this study is a replication of research where reliability and validity were not reported, this study reports the findings from the *Fraction Diagnoser* as verification of the effectiveness of an instrument of this type. The following subsections present the data for each analysis and elaborate on statistically significant findings.

Reliability Analysis

This study evaluated the reliability of *Fraction Diagnoser*, a multiple choice online assessment instrument for students in developmental mathematics at a college level. Pearson's product moment correlation coefficients were computed to assess overall and item stability between test-retest scores of 70 student participants in this study during Step 6 of the R&D cycle. Tables 9 and 10 are presentations of correlations and data.

Table 9. Descriptive Data for Test-Retest Reliability Analysis

	N	Minimum	Maximum	Mean	Std. Deviation
P1	70	0	5	2.71	2.509
P2	70	0	3	2.74	.846
P3	70	0	4	1.43	1.930
P4	70	0	4	2.17	2.007
P5	70	0	1	.46	.502
P6	70	0	3	.69	1.269
P7	70	0	2	.43	.827
P8	70	0	2	1.14	.997
P9	70	0	5	2.57	2.517
P10	70	0	1	.87	.337
P11	70	0	1	.89	.320
Score	70	1	29	16.10	5.947
P21	70	0	5	2.50	2.518
P22	70	0	3	2.96	.359
P23	70	0	4	1.66	1.985
P24	70	0	4	2.51	1.947
P25	70	0	1	.63	.487
P26	70	0	3	1.33	1.501
P27	70	0	2	.80	.987
P28	70	0	2	1.46	.896
P29	70	0	5	2.57	2.517
P210	70	0	1	.91	.282
P211	70	0	1	.90	.302
Score	70	5	31	18.23	5.520
Valid N (listwise)	70				

In Table 9, the letter P represents the item (problem) and the numbers that follow represent the trial and then the item number. For example, P28 represents the 8th test item given on the 2nd trial. These same labels will exist throughout the analysis of the data for this research question. When an item is preceded by z, this is an indication that the z-score was used.

Table 10. Correlation Coefficients for Reliability Analysis

	P1	P2	P3	P4	P5
P1	1.0000				
P2	.0263	1.0000			
P3	-.0940	.0152	1.0000		
P4	.0789	.0263	.1453	1.0000	
P5	.0938	-.0263	.1539	.1513	1.0000
P6	.0898	.0451	-.1217	.0215	.1151
P7	-.0100	.0355	.0467	.1997	.0100
P8	.0745	.1473	.1635	.1325	.2152
P9	.0262	-.0934	.0085	.0836	.0885
P10	.1616	.1873	-.0700	.2472*	.0955
P11	.0309	.0504	.0803	.1210	-.0309
SCORE1	.4818**	.1780	.3610**	.5449**	.3244**
P21	.2868*	.0000	-.3876**	.0000	-.0574
P22	.1312	.3932**	-.1615	-.1105	-.1312
P23	.0150	-.2605*	.2810*	-.1597	-.0150
P24	.0068	.1871	-.1058	.2442*	.0526
P25	.0068	.0815	-.0441	-.0526	.4087**
P26	.1254	-.1380	-.2444*	.0099	.0478
P27	.1639	.0417	-.1826	.1054	.0702
P28	.0848	.0426	-.0144	.1492	-.0203
P29	.0262	.1109	-.2897*	.1410	.0885
P210	.1288	.2708*	.0152	.1288	.0761
P211	-.1147	.2381*	-.1491	.1721	-.0765
SCORE2	.2371*	.0593	-.3684**	.1429	.0768

	P6	P7	P8	P9	P10
P6	1.0000				
P7	.2961**	1.0000			
P8	-.0098	-.0402	1.0000		
P9	.0525	.2289	.1403	1.0000	
P10	.0058	-.0074	.1848	.1391	1.0000
P11	.0886	.0782	.3240**	.0103	-.0038
SCORE1	.3019**	.3803**	.4156**	.5344**	.3101**
P21	-.1361	-.1741	-.0577	.1143	.1280
P22	.0655	.0629	-.1043	-.1170	-.0462
P23	-.0434	.0555	.0251	-.1691	-.2834*
P24	-.0040	-.0309	-.0085	.1403	.2347
P25	.0664	-.1029	.1707	-.0372	.2347
P26	.1996	.0951	-.0996	.0608	.0847
P27	.3194**	.2843**	.0589	.0934	.0523
P28	.0262	.0056	.3802**	.1139	.2454*
P29	-.0156	-.0498	.1980	.1993	.2245
P210	.0451	.0355	.2504*	.2130	.4922**
P211	-.1588	-.0580	.2887*	-.0381	.2988*
SCORE2	.0332	-.0218	.1520	.1814	.2730*

Table 10. Continued

	P11	SCORE1	P21	P22	P23
P11	1.0000				
SCORE1	.2266	1.0000			
P21	.0898	-.0121	1.0000		
P22	-.0432	-.0387	.1204	1.0000	
P23	.0286	-.0781	.0290	-.1431	1.0000
P24	.0027	.1482	.1183	-.0925	-.0737
P25	.0027	.0431	.0591	-.0925	.0463
P26	.0491	.0336	.0863	.1073	-.2827*
P27	.1100	.2232	-.1750	.0983	-.2723*
P28	.1846	.2280	.0321	-.0735	-.2040
P29	.1001	.1133	.1715	-.1170	-.1691
P210	.0504	.3250**	.0000	-.0369	-.3641**
P211	.3293**	.0137	.0476	-.0401	-.1063
SCORE2	.1788	.1750	.5996**	.0050	.0813

	P24	P25	P26	P27	P28
P24	1.0000				
P25	.1434	1.0000			
P26	-.2670*	.0306	1.0000		
P27	-.0362	.0241	.5636**	1.0000	
P28	.1292	.0627	.0915	.3016*	1.0000
P29	.3177**	.1403	-.0543	.1517	.2424*
P210	.0815	-.0241	-.0352	.1458	.2721*
P211	.3351**	.2365*	-.0863	.1750	.3320*
SCORE2	.4960**	.2748*	.2095	.2852*	.3713*

	P29	P210	P211	SCORE2
P29	1.0000			
P210	.2130*	1.0000		
P211	.2477*	.2381*	1.0000	
SCORE2	.6664**	.1152	.3615*	1.0000

** Correlation is significant at the 0.01 level (2-tailed).
* Correlation is significant at the 0.05 level (2-tailed).

Most of the correlations for the item-to-item relationships are low, and the correlations for the overall score to items are moderate, especially the correlations of each item to its particular trial (Hinkle, Wiersma, & Jus, 1998). Score 1 is statistically significant for all but two of the items, and Score 2 with all but four. The items were created to measure different skills across the objective.

To assess the test-retest reliability of *Fraction Diagnoser*, the instrument was administered to 70 student participants at a university and a community college in southeast Texas. The instrument was administered on two occasions approximately 1 month apart. Overall, test-retest reliability for *Fraction Diagnoser* was moderately high at .70 ($p < .05$) (Hinkle, Wiersma, & Jus, 1998). For individual test items, the test-retest correlations were highest for Score 2 and P29 ($r = .6664$) and lowest but still statistically significant for P25 and P211 ($r = .2365$).

In addition to the test-retest reliability coefficient, the Spearman-Brown split-half reliability coefficients were found during steps 6 and 8 of the R&D cycle. The initial *Fraction Diagnoser* instrument in step 6 yielded a split-half reliability coefficient of .73. Then, the instrument in step 8 had a Spearman-Brown reliability of .82. According to Huck (2000), both reliability coefficients would be considered high.

Convergent and Discriminant Validity

The latest revision of *Fraction Diagnoser* is an 18 item online instrument designed to assess both skills and appropriate *facets* of thought. The items are scored using a skills checklist and an adapted rubric from the California Mathematics Project. The descriptive data for *Fraction Diagnoser* along with the participants’ scores on the THEA test and THEA number subgroup can be seen in Table 11.

The convergent construct validity was established by examining correlations using Pearson's *r* between the assessment item scores of the *Fraction Diagnoser* and the diagnostic items with which they were expected to associate (see Table 12). It was anticipated that the assessment items would correlate significantly with the following or appropriate diagnostic items. Conclusively, if there is evidence that these items correlate significantly with one another, this provides evidence for convergent construct validity.

Also, because of the small scope of the content area of *Fraction Diagnoser*, it was expected that items and score would not correlate strongly or significantly with the overall THEA Score or THEA number subgroup. This was theorized because the *Fraction Diagnoser* assesses only a small content scope of the previously mentioned tests. It was expected that scores would correlate more strongly with the THEA Number subgroup, which in theory is more content similar. This information, if found, would lend support to the existence of discriminant construct validity.

Table 11. Descriptive Statistics of Correlation Items

	Mean	Std. Deviation	N
Q1P	.23	1.057	449
Q2P	3.88	1.537	449
Q3P	2.47	1.144	449
Q4P	4.27	1.758	449
Q5P	1.47	1.931	449
Q6P	3.26	1.843	449
Q7P	1.83	1.995	449
Q8P	3.05	1.125	449
Q9P	.94	.246	449
Q10P	4.14	2.323	449
Q11P	1.20	1.472	449
Q12P	.68	.947	449
Q13P	.73	.964	449
Q14P	3.72	1.818	449
Q15P	3.14	2.419	449
Q16P	3.01	1.618	449
Q17P	.94	.242	449
Q18P	.87	.338	449
FD Score	39.83	9.524	449
MTH	196.39	24.065	449
FMATH	45.63	18.390	448

Table 12. Validity Correlations of *Fraction Diagnoser* to THEA Number Subgroup

	Q1P	Q2P	Q3P	Q4P	Q5P
Q1P	1.0000				
Q2P	.1065*	1.0000			
Q3P	.1026*	.0177	1.0000		
Q4P	.0014	.0228	.2490**	1.0000	
Q5P	.2029**	.0679	.0133	.0176	1.0000
Q6P	.1005*	.2250**	.0084	.1518**	.1934**
Q7P	.0729	-.0492	-.1062*	-.0689	.0731
Q8P	.1402**	.1691**	-.0831	.2189**	-.0184
Q9P	-.0275	.1451	.0449	.2063	.0692
Q10P	.0866	.1931**	.1113*	.1104*	.0893
Q11P	.1641**	-.0113	.0546	.1475**	.0668
Q12P	.1770**	.0611	.0449	.1331**	.1604**
Q13P	.1396**	-.0082	-.0031	.0165	.0093
Q14P	.1387**	.2074**	-.0259	.0562	-.0254
Q15P	.0827	-.0651	.1301**	.1146*	.2026**
Q16P	.1353**	.1462**	-.0179	.1841**	.0366
Q17P	.0573	-.0139	.0015	.0298	.1589**
Q18P	-.0073	.0001	.0623	.0496	.0921
FDS	.3820**	.3481**	.2206**	.4060**	.4021**
MTH	.0498	.0484	.0533	-.0961	.1438
	Q1P	Q2P	Q3P	Q4P	Q5P
FMATH	-.0812	.0353	.0110	-.0132	.1256

	Q6P	Q7P	Q8P	Q9P	Q10P
Q6P	1.0000				
Q7P	-.0567	1.0000			
Q8P	.2116**	.1018*	1.0000		
Q9P	.1113*	.0402	.1412**	1.0000	
Q10P	.0928	.1203**	.1786**	.2660**	1.0000
Q11P	.0056	.0686	.0966	.0479	-.0455
Q12P	.0836	.1350**	.1565**	.0149	.0097
Q13P	.0566	.0818	.1306**	-.0462	-.0338
Q14P	.2603**	.0664	.3668**	.0295	.0834
Q15P	.0084	-.0502	-.0061	.0048	-.0133
Q16P	.2419**	.0534	.2096**	.0583	.0025
Q17P	-.0435	-.0417	.0282	.1195	-.0201
Q18P	.0804	.0512	.0941	.0049	.0407
FDS	.4569**	.2721**	.4486**	.2392**	.4118**
MTH	-.0186	.0327	-.0350	.0952	.0058
FMATH	.0086	.0605	-.0682	-.0236	-.0352

Table 12. Continued

	Q11P	Q12P	Q13P	Q14P	Q15P
Q11P	1.0000				
Q12P	.5174**	1.0000			
Q13P	.2735**	.4226**	1.0000		
Q14P	.1103*	.2473**	.2030**	1.0000	
Q15P	.1163*	.2244**	.0807	.0145	1.0000
Q16P	.0806	.1780**	.1372**	.1593**	.1378**
Q17P	.0977	.1256**	.0419	-.2021**	.2411**
Q18P	.0751	.0822	-.0210	-.0017	.0156
FDS	.3833**	.5197**	.3207**	.4578**	.3985**
MTH	-.0071	-.0265	-.0561	.0265	.0197
FMATH	-.0559	-.0516	-.0079	-.0592	-.0151

	Q16P	Q17P	Q18P	FDS	MTH
Q16P	1.0000				
Q17P	.0363	1.0000			
Q18P	-.0457	.0631	1.0000		
FDS	.4410**	.1146**	.1325**	1.0000	
MTH	-.1248**	.0274	.1527**	.0197	1.0000
FMATH	-.1365**	.0043	.0848	-.0316	.5326**

	FMATH
FMATH	1.0000

** Correlation is significant at the 0.01 level (2-tailed).
* Correlation is significant at the 0.05 level (2-tailed).

As expected, all of the diagnostic items for the *Fraction Diagnoser* (FD) correlated significantly with their assessment items. All were found to be significant at the .01 level with the exception of 1 to 2, 7 to 8, and 11 to 14 which were significant at the .05 level. Specifically, the assessment to diagnostic item correlation coefficients were: 1 to 2 (.1065*); 3 to 4 (.2490**); 5 to 6 (.1934**); 7 to 8 (.1018*); 9 to 10 (.2660**); 11 to 14 (.1103*); 12 to 14 (.2473**); 13 to 14 (.2030**); 15 to 16 (.1378**).

Also, the correlation between the FD Score and the THEA test score was expectedly low (.0197) and was higher for the FD to THEA number subgroup (-.0316), lending support to a discriminant construct validity.

Predictive Validity

To further address the second research question, a regression was used to determine the predictive value of the *Fraction Diagnoser.* The data used in the regression was from R&D Steps 6 and 8, where the test items were used as the predictive variables and the THEA Fundamentals score was used as the criterion variable. Multicollinearity was not a concern in this study because there were no strong correlations between the item scores in *Fraction Diagnoser*. Multiple linear regression analysis produced the following full formula equation for predicting the THEA Fundamentals score:

THEA Fund = 44.23 - 1.548(Item1) + .999(Item2) + .319(Item3) + .377(Item4) + 1.357(Item5) + .228(Item6) + .824(Item7) - .586(Item8) - 2.046(Item9) - .486(Item10) - .480(Item11) - 1.058(Item12) + 1.058(Item13) - .270(Item14) - .029(Item15) - 1.544(Item16) - .086(Item17) + 3.80(Item18).

The analysis had statistically significant findings for $p<.05$ (see Table 13).

Table 13. ANOVA Table for Regressional Analysis of *Fraction Diagnoser*

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	10477.218	18	582.068	1.779	.025(a)
	Residual	140689.530	430	327.185		
	Total	151166.748	448			

a Predictors: (Constant), Q18P, Q2P, Q13P, Q4P, Q17P, Q7P, Q1P, Q10P, Q5P, Q16P, Q3P, Q11P, Q9P, Q15P, Q14P, Q6P, Q8P, Q12P
b Dependent Variable: FMATH

In addition, *Fraction Diagnoser* results produced an R^2 or effect size of 13.7% using a univariate test of between-subjects effects with ά = .05. Therefore, the linear combination of the predictor variables studied accounted for less than 15% of the variance in the THEA number (Huck, 2000). Also, there were only two statistically significant Beta weights. The two statistically significant items were numbers 5 and 16, where $p<.01$ on both items. Although not strongly, the findings from the analysis support the hypothesis that the *Fraction Diagnoser* could be used to predict the THEA number subgroup performance.

Research Question 3 – Validation: (Within subjects) What Kinds of Student Information Did the Fraction Diagnoser Provide to Describe Student Growth Toward Mastery in MUL?

Fraction Diagnoser provides both summative and formative data of student progress. The online assessment instrument was built to score student items related to both skills and reasoning. Scoring of the assessment items was done using a team of experts to score skills items related to a skills checklist (Appendix E), while scoring of the diagnostic items was done by the same experts using a rubric adapted from the California Mathematics Council's Rubric for Open-Ended Questions (Appendix F). In the end, *Fraction Diagnoser* sums each item score to provide a total MUL score for a student.

Fraction Diagnoser Formative Data Presentation

This section of the chapter will explain the data that *Fraction Diagnoser* provides to assess and document individual student growth. In addition to the data concerning individual students, *Fraction Diagnoser* provides overall class information in the form of item demographics (see Figures 21-25). The following figures are used to give a step by step detailed description of what a teacher should do in order to access and to use the instrument from any personal computer with internet access. You should follow the reading very carefully because some the steps are numbered and some are described in the text.

A teacher or researcher must follow the following steps to get to the *Fraction Diagnoser* Database:

1. Go to the website www.boocent.com/admin (see Figure 12)

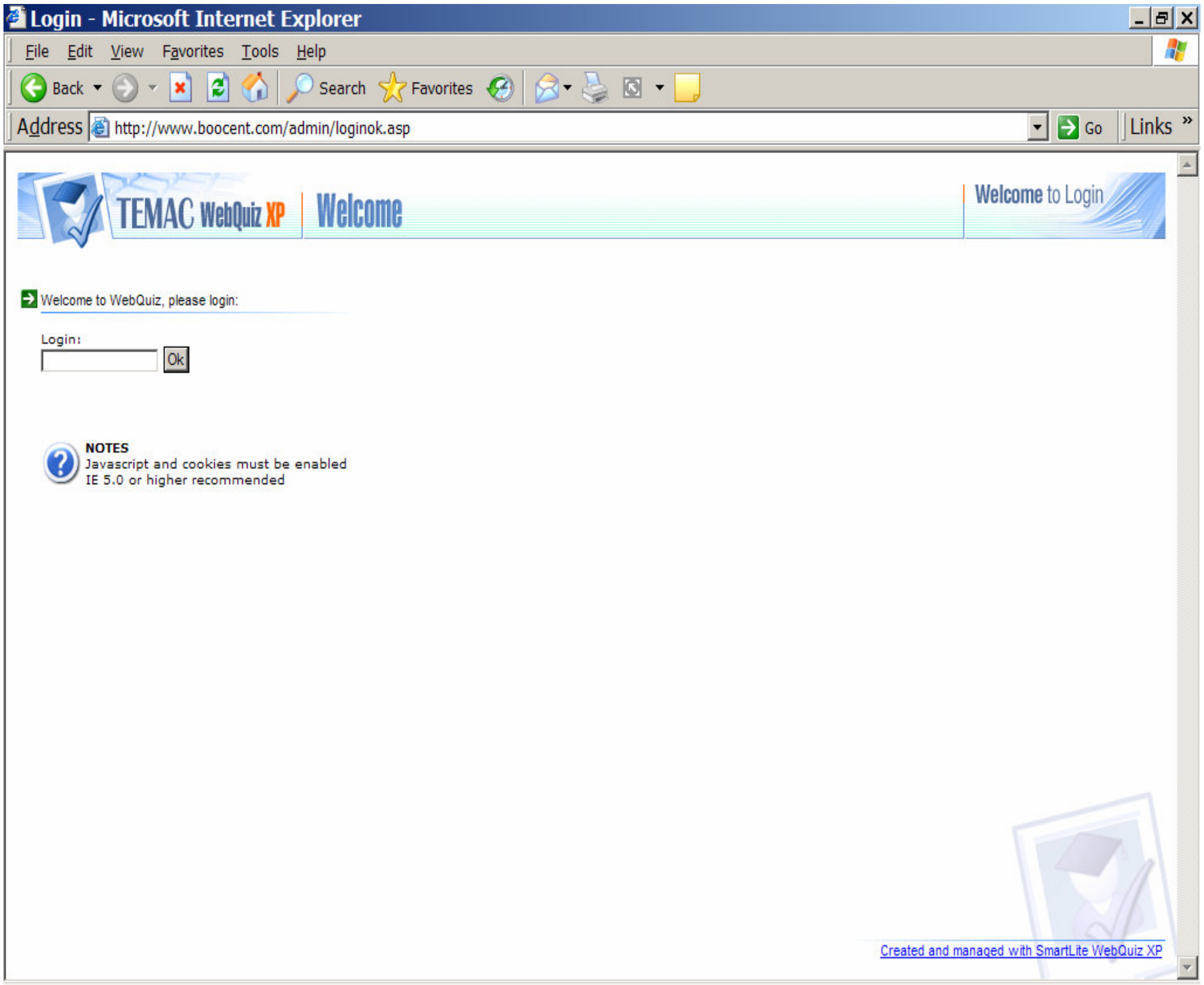
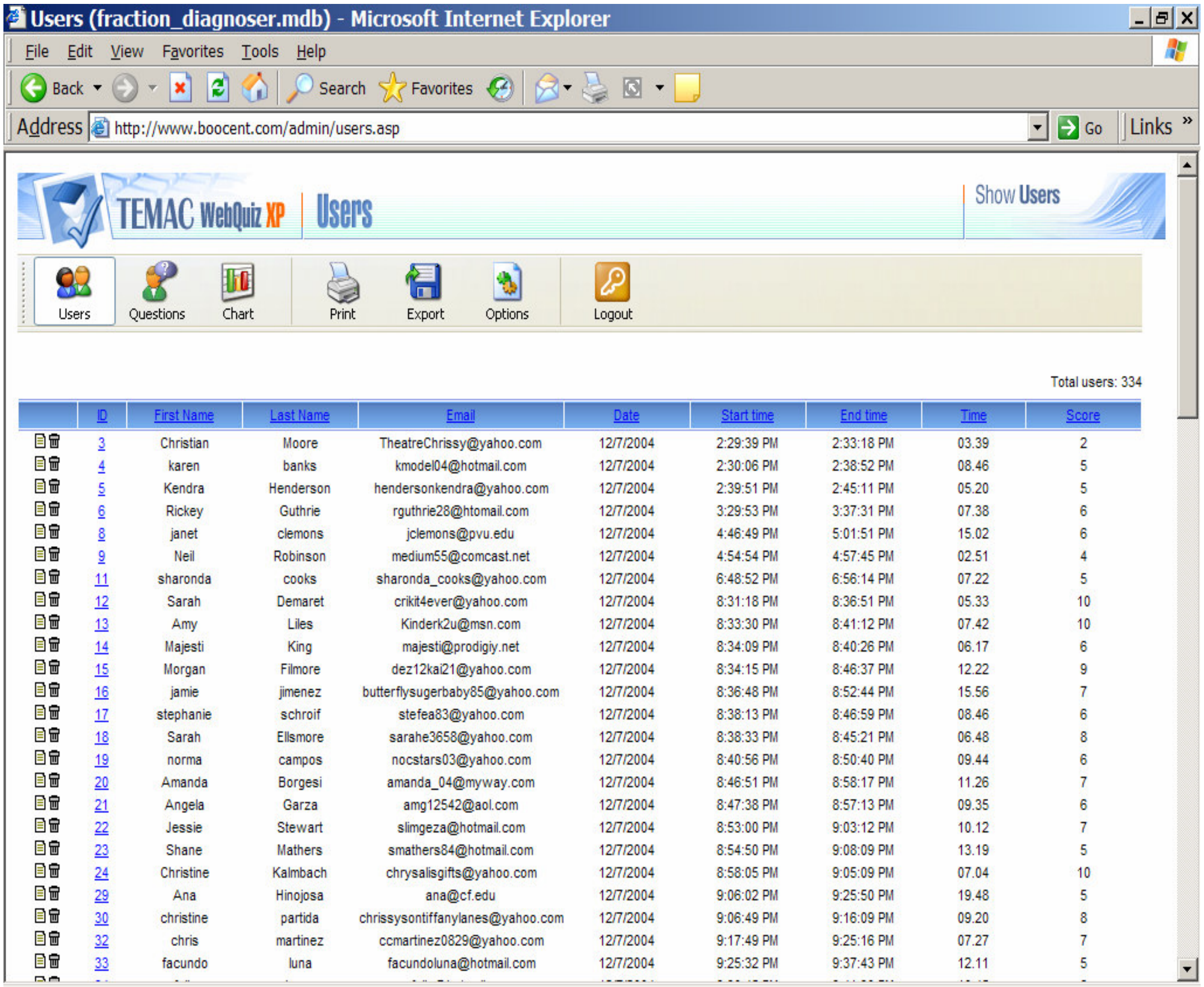


Figure 12. The login webpage for *Fraction Diagnoser* student database.

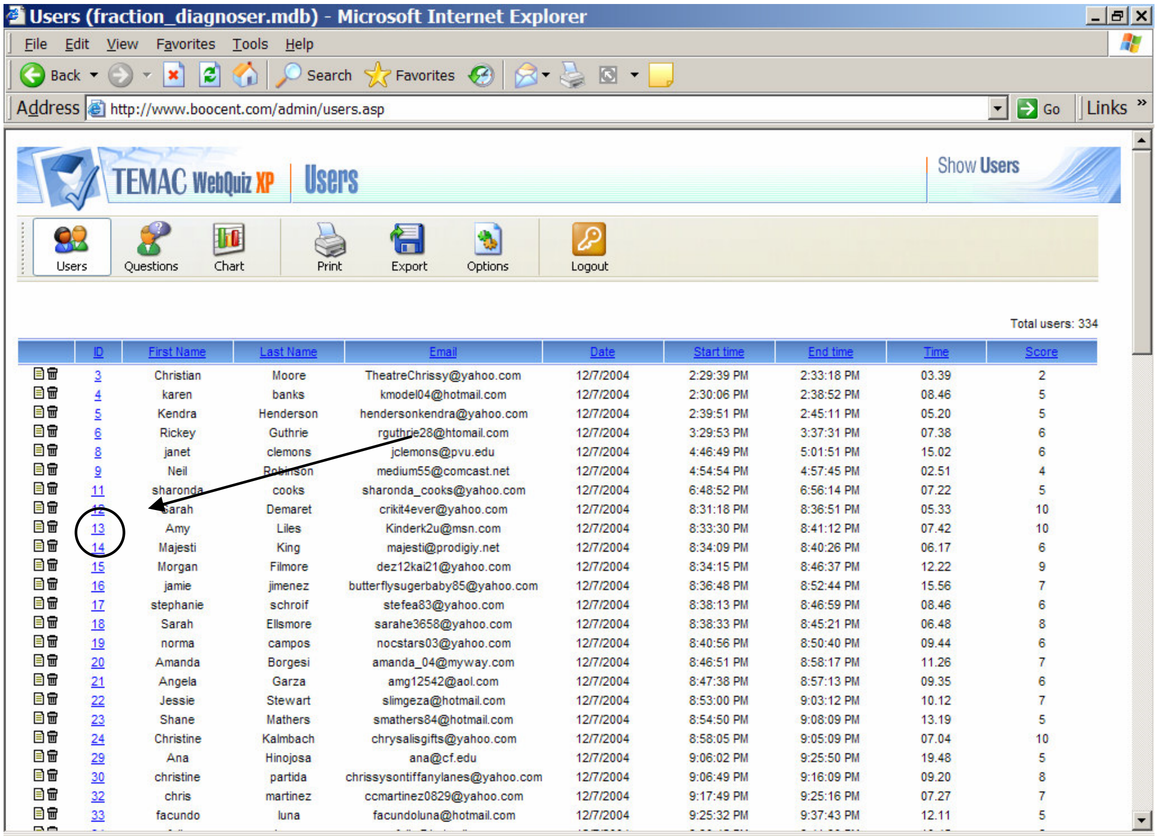
2. Enter the password to get to the webpage shown in Figure 13.



ID	First Name	Last Name	Email	Date	Start time	End time	Time	Score
3	Christian	Moore	TheatreChrissy@yahoo.com	12/7/2004	2:29:39 PM	2:33:18 PM	03.39	2
4	karen	banks	kmodel04@hotmail.com	12/7/2004	2:30:06 PM	2:38:52 PM	08.46	5
5	Kendra	Henderson	hendersonkendra@yahoo.com	12/7/2004	2:39:51 PM	2:45:11 PM	05.20	5
6	Rickey	Guthrie	rguthrie28@htomail.com	12/7/2004	3:29:53 PM	3:37:31 PM	07.38	6
8	janet	clemons	jclemons@pvu.edu	12/7/2004	4:46:49 PM	5:01:51 PM	15.02	6
9	Neil	Robinson	medium55@comcast.net	12/7/2004	4:54:54 PM	4:57:45 PM	02.51	4
11	sharonda	cooks	sharonda_cooks@yahoo.com	12/7/2004	6:48:52 PM	6:56:14 PM	07.22	5
12	Sarah	Demaret	crikit4ever@yahoo.com	12/7/2004	8:31:18 PM	8:36:51 PM	05.33	10
13	Amy	Liles	Kinderk2u@msn.com	12/7/2004	8:33:30 PM	8:41:12 PM	07.42	10
14	Majesti	King	majesti@prodigiy.net	12/7/2004	8:34:09 PM	8:40:26 PM	06.17	6
15	Morgan	Filmore	dez12kai21@yahoo.com	12/7/2004	8:34:15 PM	8:46:37 PM	12.22	9
16	jamie	jimenez	butterflysugerbaby85@yahoo.com	12/7/2004	8:36:48 PM	8:52:44 PM	15.56	7
17	stephanie	schroif	stefea83@yahoo.com	12/7/2004	8:38:13 PM	8:46:59 PM	08.46	6
18	Sarah	Ellsmore	sarahe3658@yahoo.com	12/7/2004	8:38:33 PM	8:45:21 PM	06.48	8
19	norma	campos	nocstars03@yahoo.com	12/7/2004	8:40:56 PM	8:50:40 PM	09.44	6
20	Amanda	Borgesi	amanda_04@myway.com	12/7/2004	8:46:51 PM	8:58:17 PM	11.26	7
21	Angela	Garza	amg12542@aol.com	12/7/2004	8:47:38 PM	8:57:13 PM	09.35	6
22	Jessie	Stewart	slimgeza@hotmail.com	12/7/2004	8:53:00 PM	9:03:12 PM	10.12	7
23	Shane	Mathers	smathers84@hotmail.com	12/7/2004	8:54:50 PM	9:08:09 PM	13.19	5
24	Christine	Kalmbach	chrysalisgifts@yahoo.com	12/7/2004	8:58:05 PM	9:05:09 PM	07.04	10
29	Ana	Hinojosa	ana@cf.edu	12/7/2004	9:06:02 PM	9:25:50 PM	19.48	5
30	christine	partida	chrissysontiffanylanes@yahoo.com	12/7/2004	9:06:49 PM	9:16:09 PM	09.20	8
32	chris	martinez	ccmartinez0829@yahoo.com	12/7/2004	9:17:49 PM	9:25:16 PM	07.27	7
33	facundo	luna	facundoluna@hotmail.com	12/7/2004	9:25:32 PM	9:37:43 PM	12.11	5

Figure 13. The home webpage of the *Fraction Diagnoser* student database.

It is at this webpage that *Fraction Diagnoser* allows a teacher/researcher to evaluate individual student data or entire class data. For example, if a teacher/researcher wanted to look at the specific student with ID number 13, they would have to click on the blue link number 13 (see Figure 14) and individual student data would appear (see Figure 15).



ID	First Name	Last Name	Email	Date	Start time	End time	Time	Score
3	Christian	Moore	TheatreChrissy@yahoo.com	12/7/2004	2:29:39 PM	2:33:18 PM	03.39	2
4	karen	banks	kmodel04@hotmail.com	12/7/2004	2:30:06 PM	2:38:52 PM	08.46	5
5	Kendra	Henderson	hendersonkendra@yahoo.com	12/7/2004	2:39:51 PM	2:45:11 PM	05.20	5
6	Rickey	Guthrie	rguthrie28@htomail.com	12/7/2004	3:29:53 PM	3:37:31 PM	07.38	6
8	janet	clemons	jclemons@pvu.edu	12/7/2004	4:46:49 PM	5:01:51 PM	15.02	6
9	Neil	Robinson	medium55@comcast.net	12/7/2004	4:54:54 PM	4:57:45 PM	02.51	4
11	sharonda	cooks	sharonda_cooks@yahoo.com	12/7/2004	6:48:52 PM	6:56:14 PM	07.22	5
12	Sarah	Demaret	crikit4ever@yahoo.com	12/7/2004	8:31:18 PM	8:36:51 PM	05.33	10
13	Amy	Liles	Kinderk2u@msn.com	12/7/2004	8:33:30 PM	8:41:12 PM	07.42	10
14	Majesti	King	majesti@prodigiy.net	12/7/2004	8:34:09 PM	8:40:26 PM	06.17	6
15	Morgan	Filmore	dez12kai21@yahoo.com	12/7/2004	8:34:15 PM	8:46:37 PM	12.22	9
16	jamie	jimenez	butterflysugerbaby85@yahoo.com	12/7/2004	8:36:48 PM	8:52:44 PM	15.56	7
17	stephanie	schroif	stefea83@yahoo.com	12/7/2004	8:38:13 PM	8:46:59 PM	08.46	6
18	Sarah	Ellsmore	sarahe3658@yahoo.com	12/7/2004	8:38:33 PM	8:45:21 PM	06.48	8
19	norma	campos	nocstars03@yahoo.com	12/7/2004	8:40:56 PM	8:50:40 PM	09.44	6
20	Amanda	Borgesi	amanda_04@myway.com	12/7/2004	8:46:51 PM	8:58:17 PM	11.26	7
21	Angela	Garza	amg12542@aol.com	12/7/2004	8:47:38 PM	8:57:13 PM	09.35	6
22	Jessie	Stewart	slimgeza@hotmail.com	12/7/2004	8:53:00 PM	9:03:12 PM	10.12	7
23	Shane	Mathers	smathers84@hotmail.com	12/7/2004	8:54:50 PM	9:08:09 PM	13.19	5
24	Christine	Kalmbach	chrysalisgifts@yahoo.com	12/7/2004	8:58:05 PM	9:05:09 PM	07.04	10
29	Ana	Hinojosa	ana@cf.edu	12/7/2004	9:06:02 PM	9:25:50 PM	19.48	5
30	christine	partida	chrissysontiffanylanes@yahoo.com	12/7/2004	9:06:49 PM	9:16:09 PM	09.20	8
32	chris	martinez	ccmartinez0829@yahoo.com	12/7/2004	9:17:49 PM	9:25:16 PM	07.27	7
33	facundo	luna	facundoluna@hotmail.com	12/7/2004	9:25:32 PM	9:37:43 PM	12.11	5

Figure 14. A picture showing what a teacher/researcher should click on in order to see individual student data for the user 13.

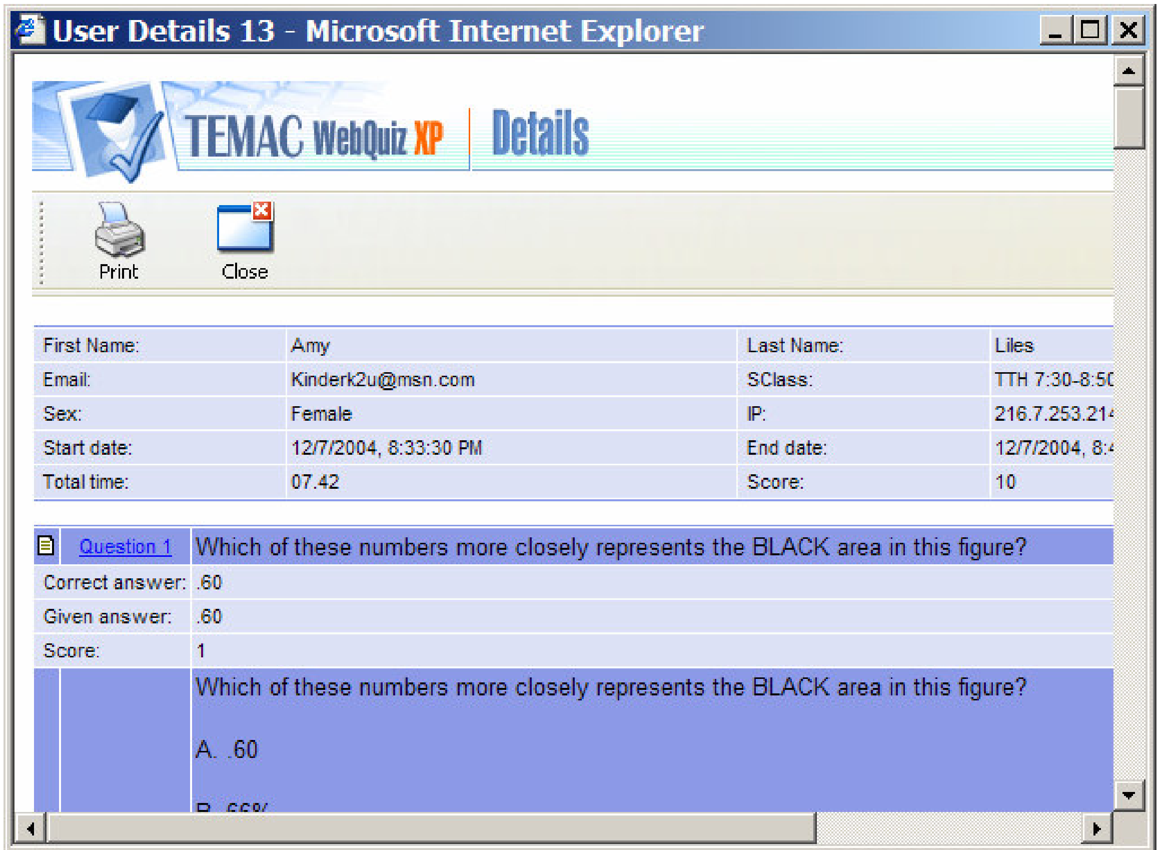


Figure 15. A picture of the grade page from *Fraction Diagnoser* that shows user 13's individual item responses.

Note that the *Fraction Diagnoser* gives general information as to the student's score, time and date when the instrument was taken, total elapsed time used to take the instrument, and the class or teacher they may have taken the test for. From this point a teacher/researcher can scroll down and see each item and how that student responded to that particular question (see Figures 15-19).

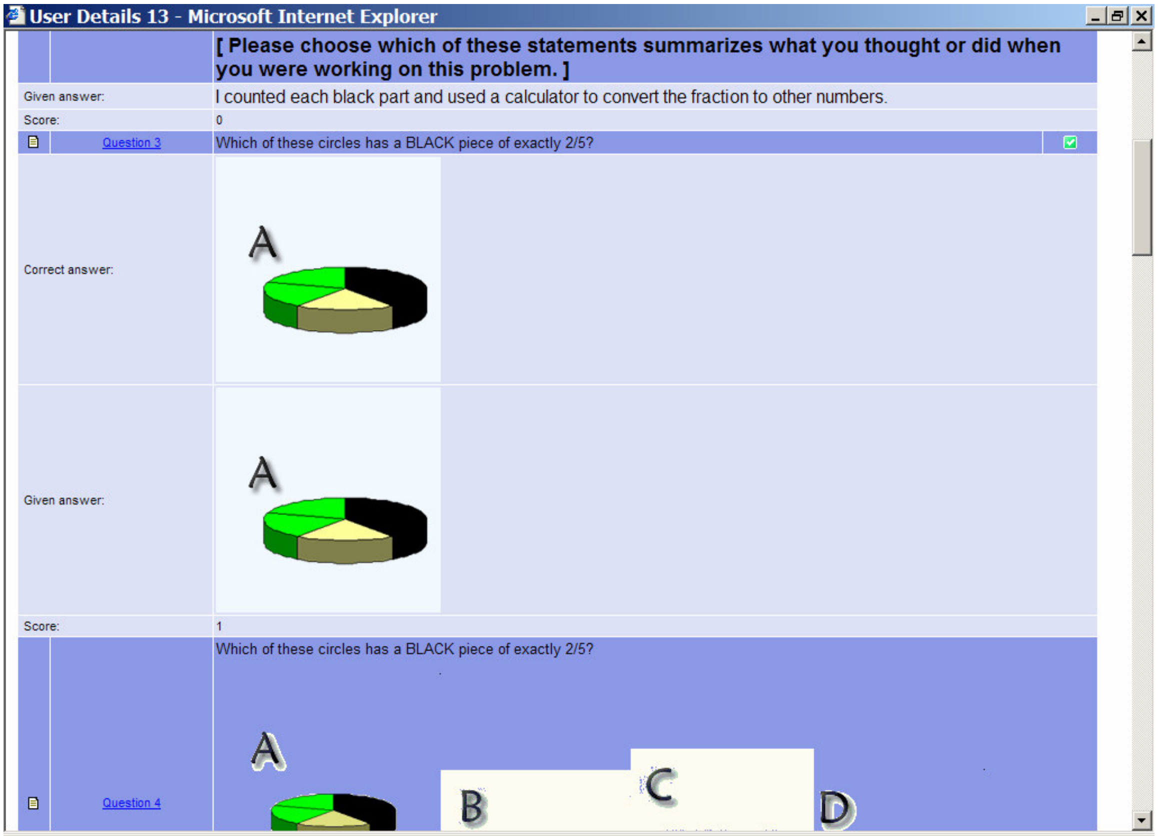


Figure 16. User 13's responses to items 3 and 4 in *Fraction Diagnoser.*

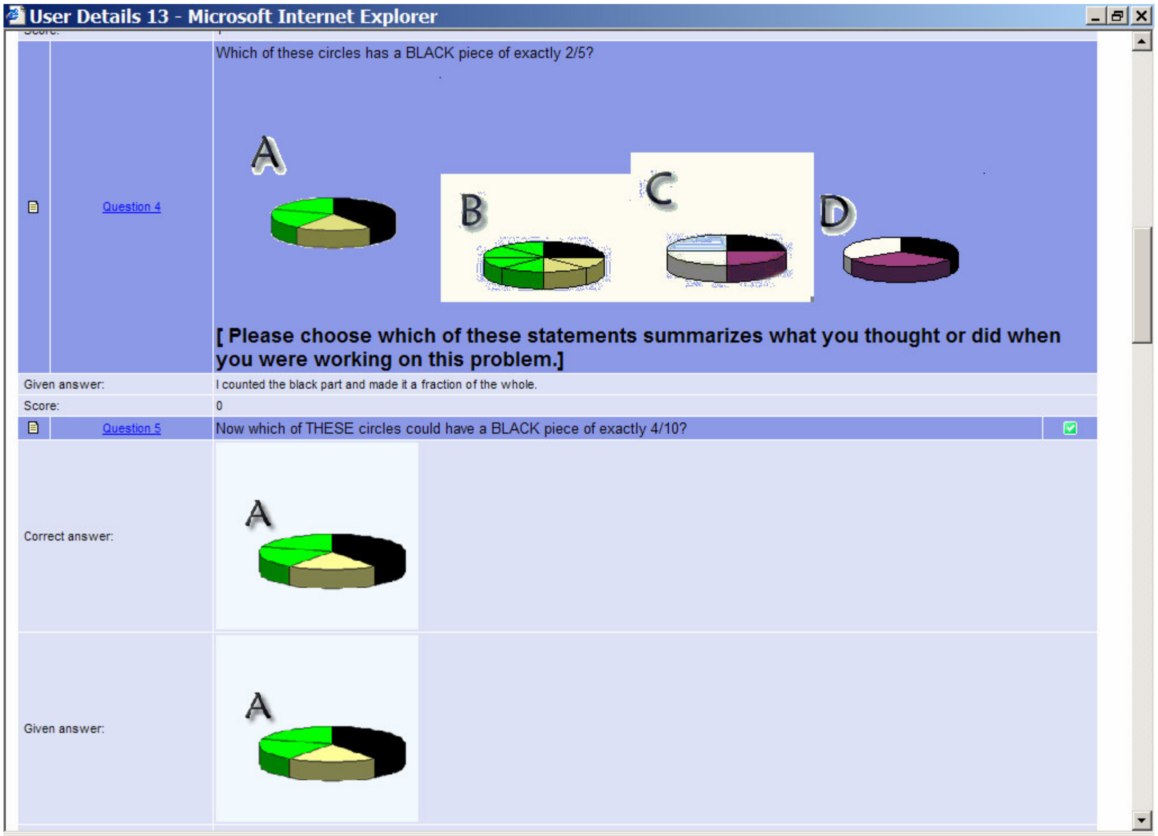


Figure 17. User 13's responses to items 4 and 5 in *Fraction Diagnoser.*

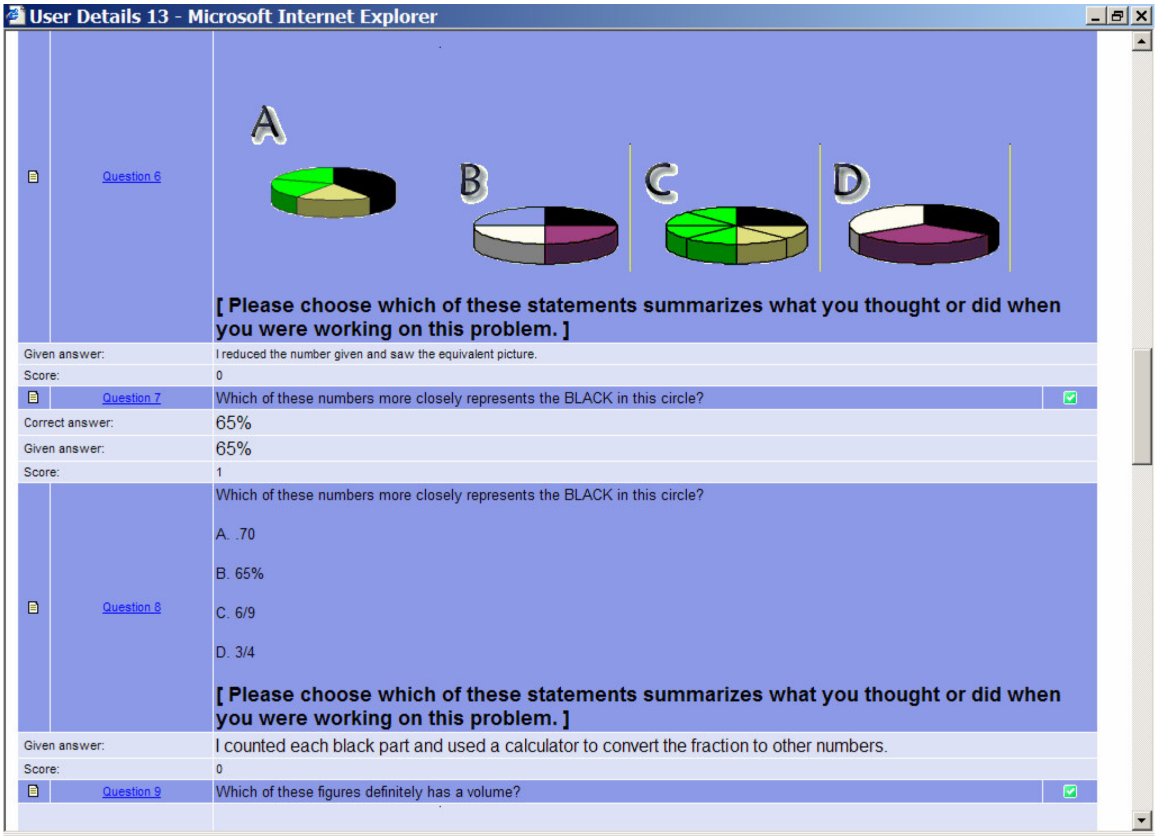


Figure 18. User 13's responses to items 6, 7, and 8 in *Fraction Diagnoser.*

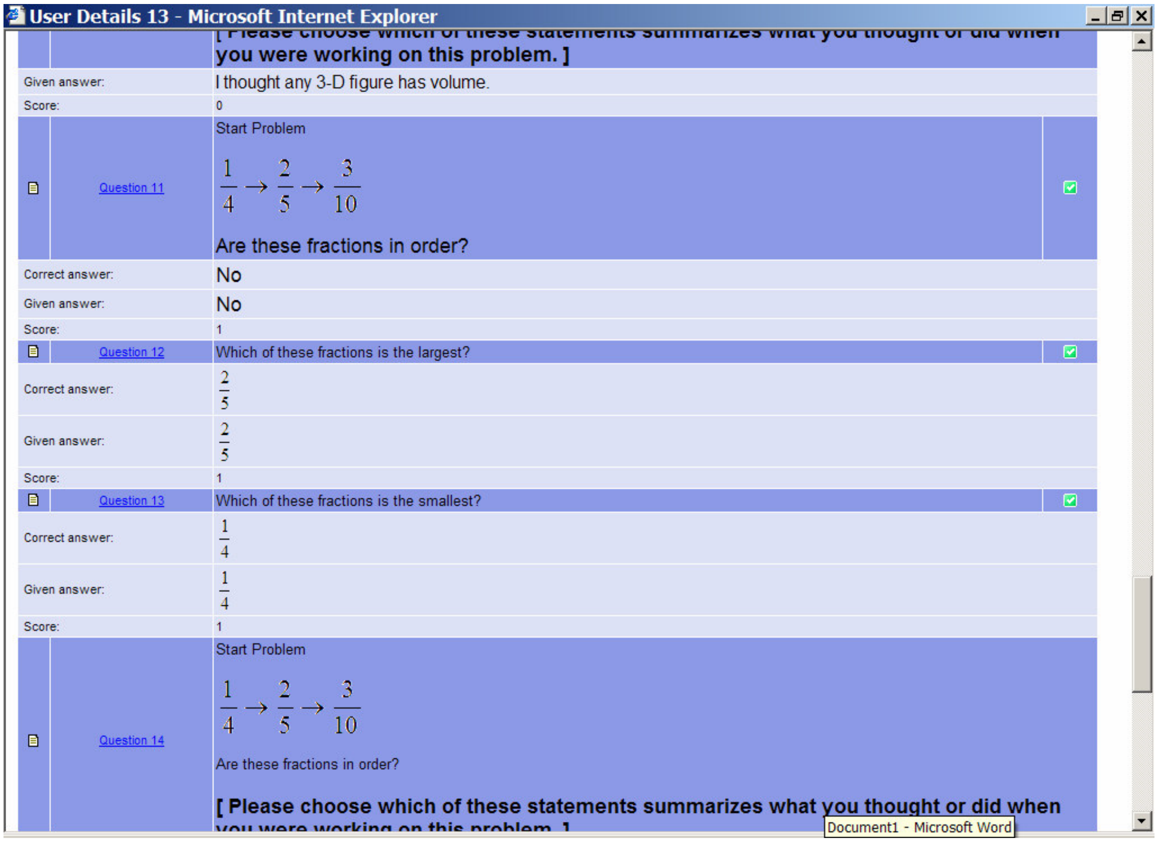


Figure 19. User 13's responses to items 12 and 13 in *Fraction Diagnoser.*

By looking at a student's individual score at the time that they used the instrument, a teacher can track progress of individual student data. Also, if the teacher/researcher wanted to specifically chart the information on individual students, *Fraction Diagnoser* does not chart individual progress, but the data from the instrument could be exported to a database or spreadsheet that does (see Figure 20).

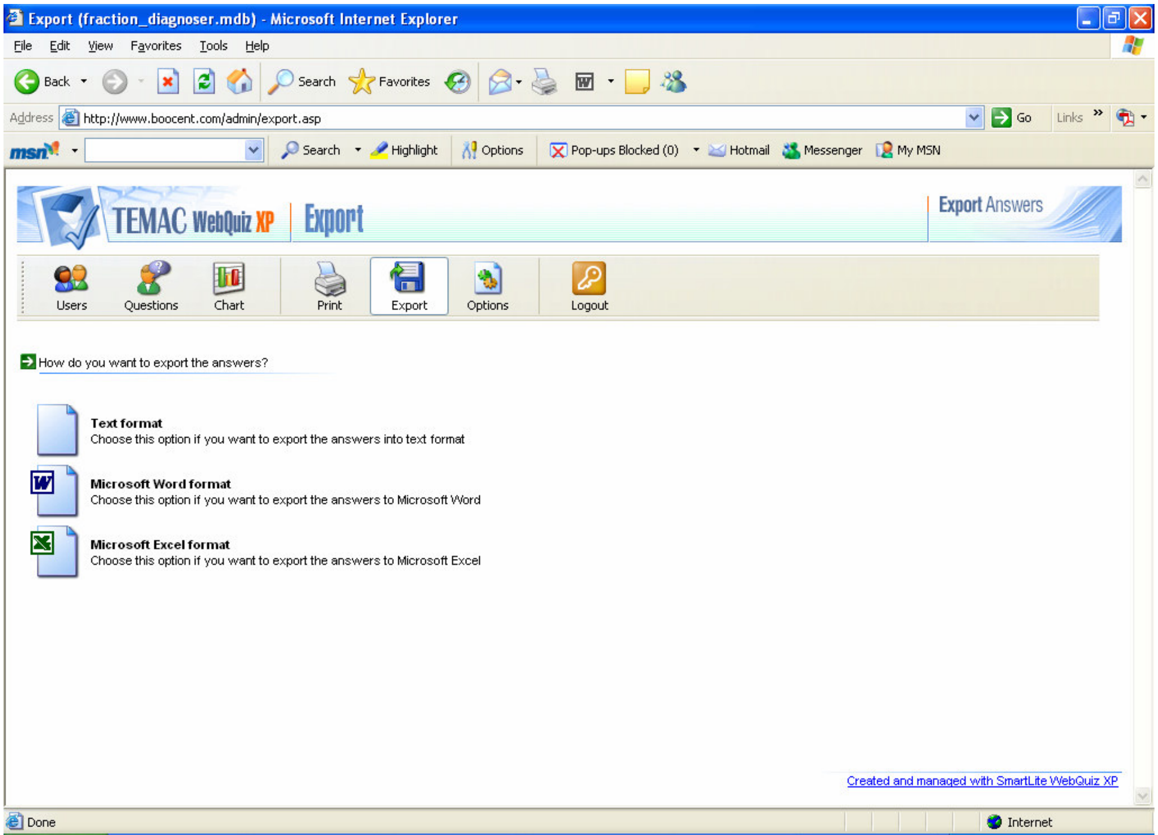


Figure 20. The *Fraction Diagnoser* export page that allows teachers/researchers to send *Fraction Diagnoser* information to an outside database.

As seen in Figure 20, the information from *Fraction Diagnoser* could be exported to either a text or spreadsheet format for further manipulation and/or analysis.

Fraction Diagnoser Summative Data Presentation

Fraction Diagnoser also provides efficient feedback as to class understanding or summative data toward mastery of MUL. The online program gives an overall look at student data that can be used to evaluate the group and their misconceptions. After following the steps to get to the data, a teacher/

researcher can click the “Users” button on the *Fraction Diagnoser* Data webpage (see Figure 21).

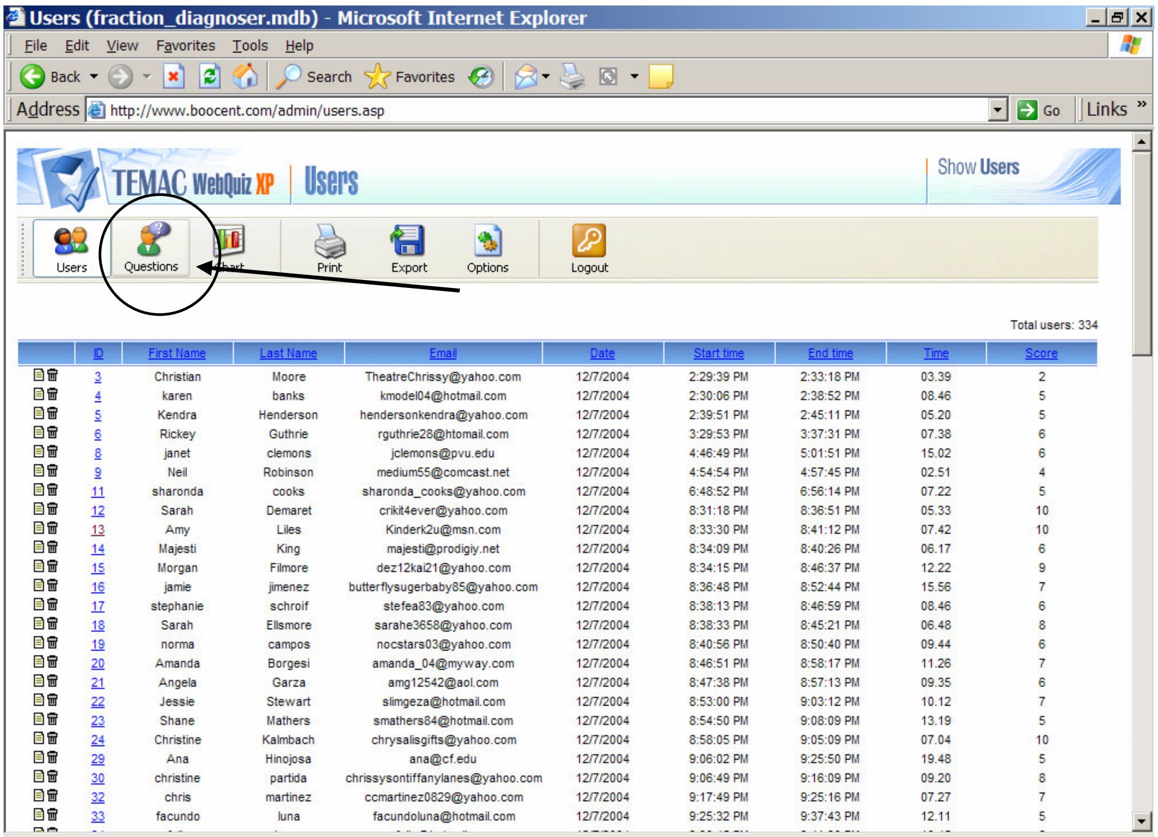


Figure 21. How to access summative information concerning all users from the *Fraction Diagnoser* data webpage.

A click on the “Questions” button of the *Fraction Diagnoser* program will take the teacher/researcher to a webpage that gives information concerning all of the questions in the database (see Figure 22).

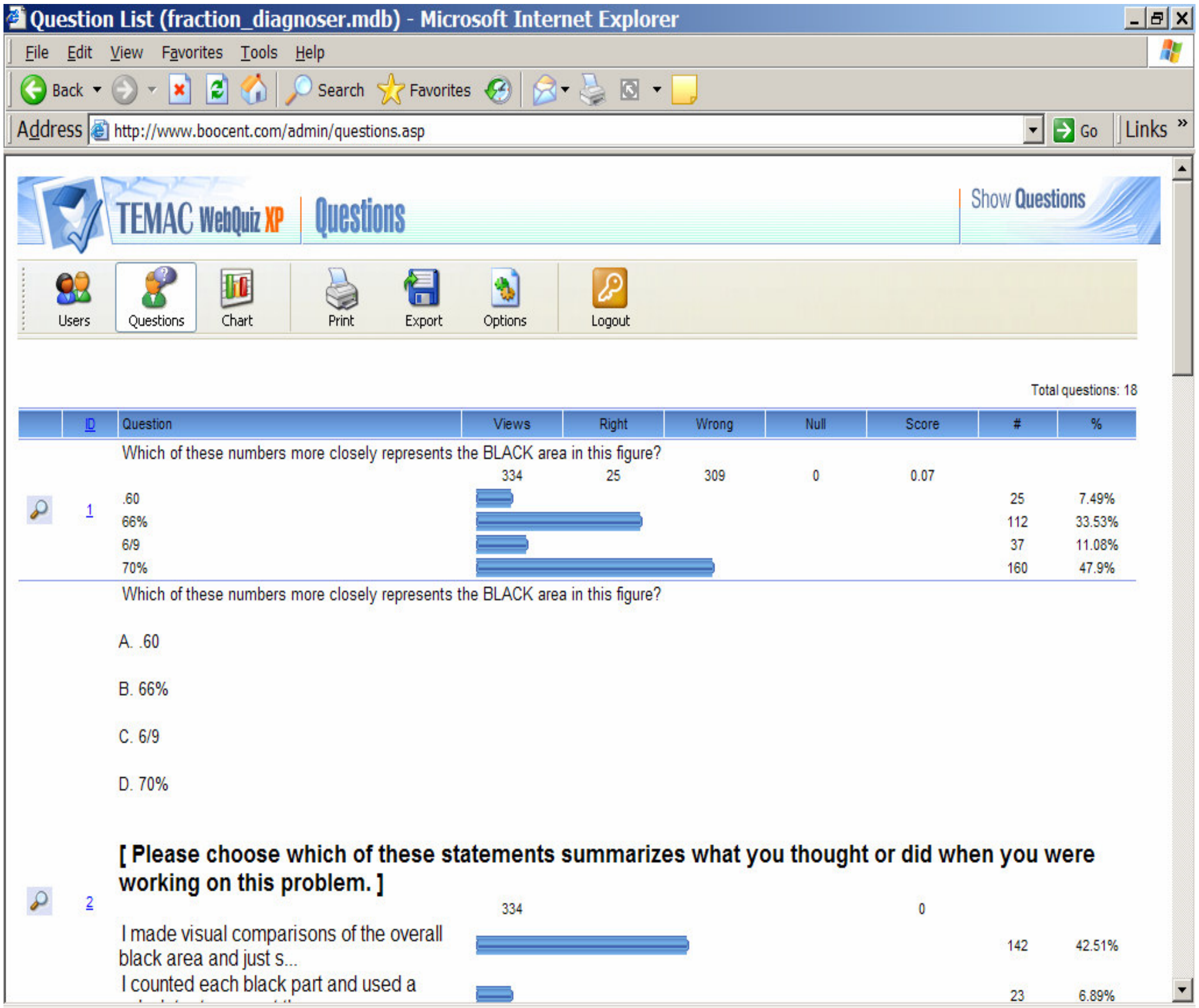


Figure 22. Overall summative data concerning questions 1 and half of 2 (cut off) from *Fraction Diagnoser.*

Fraction Diagnoser shows that although the correct response for question number 1 was choice A, .60, only 7.49% (or 25) of the 334 students actually selected that response. It also shows in the Diagnostic Question for that item (#2) that the correct responses coincide almost perfectly with the 25 students who chose the highest *facet* of understanding for that question (see Figure 23).

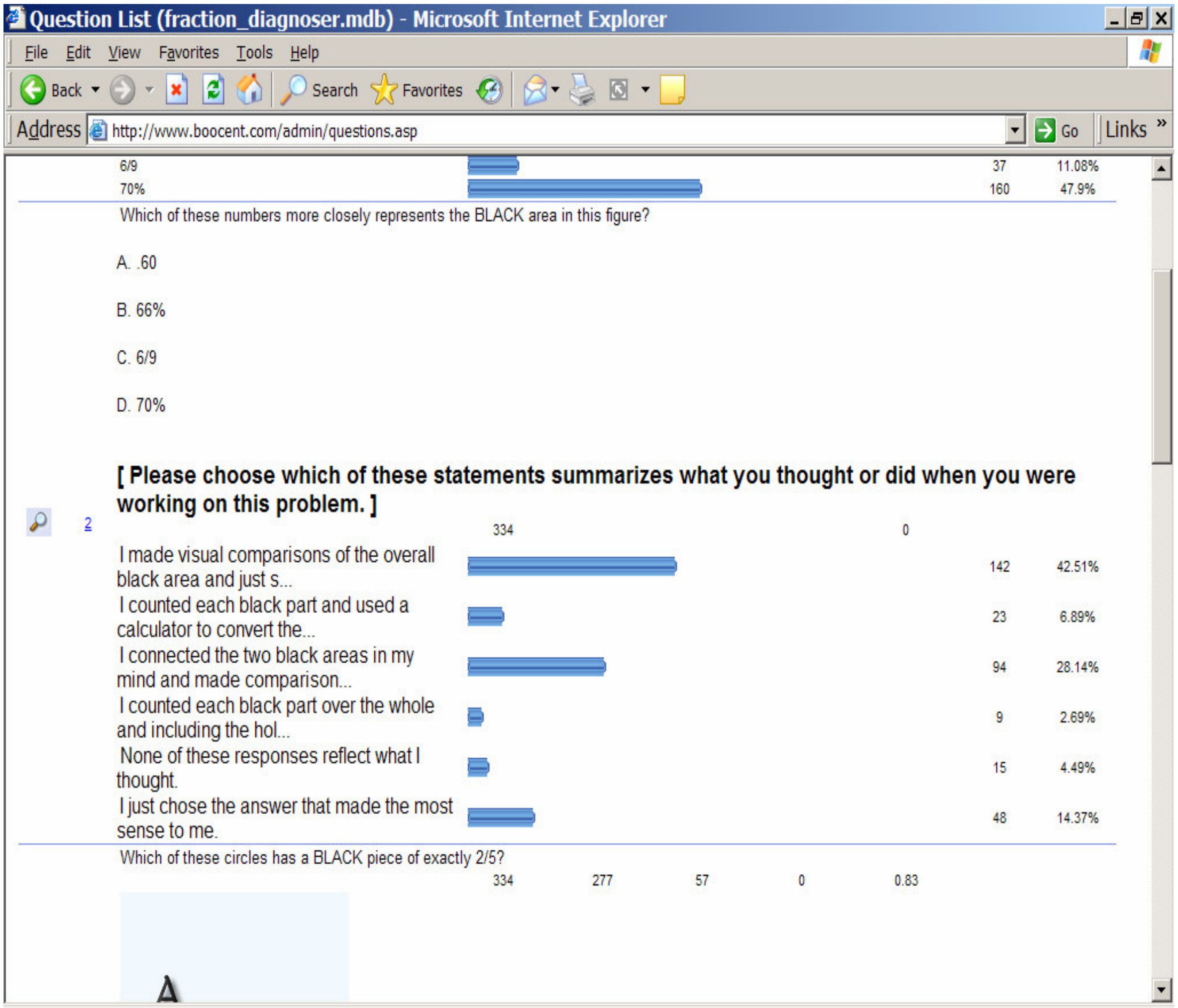


Figure 23. The picture of the *Fraction Diagnoser* questions page for the entire item number 2, showing the *facets* chosen for the response to assessment item 1 (Figure 19 only showed half).

Fraction Diagnoser also showed evidence of inconsistencies in some responses. Minstrell (2002) talked about how in the design of *Diagnoser* it was necessary to allow students to go back and change responses to allow for ongoing metacognition, so *Fraction Diagnoser* was built with that feature. However, the addition of this feature has led to some inconclusive data. For example, in some assessment items students would choose responses and then, after reading the *facets* in the diagnostic items, realize that maybe their answers could have been thought about differently. *Fraction Diagnoser* data shows evidence of these phenomena (see Figures 24 & 25).

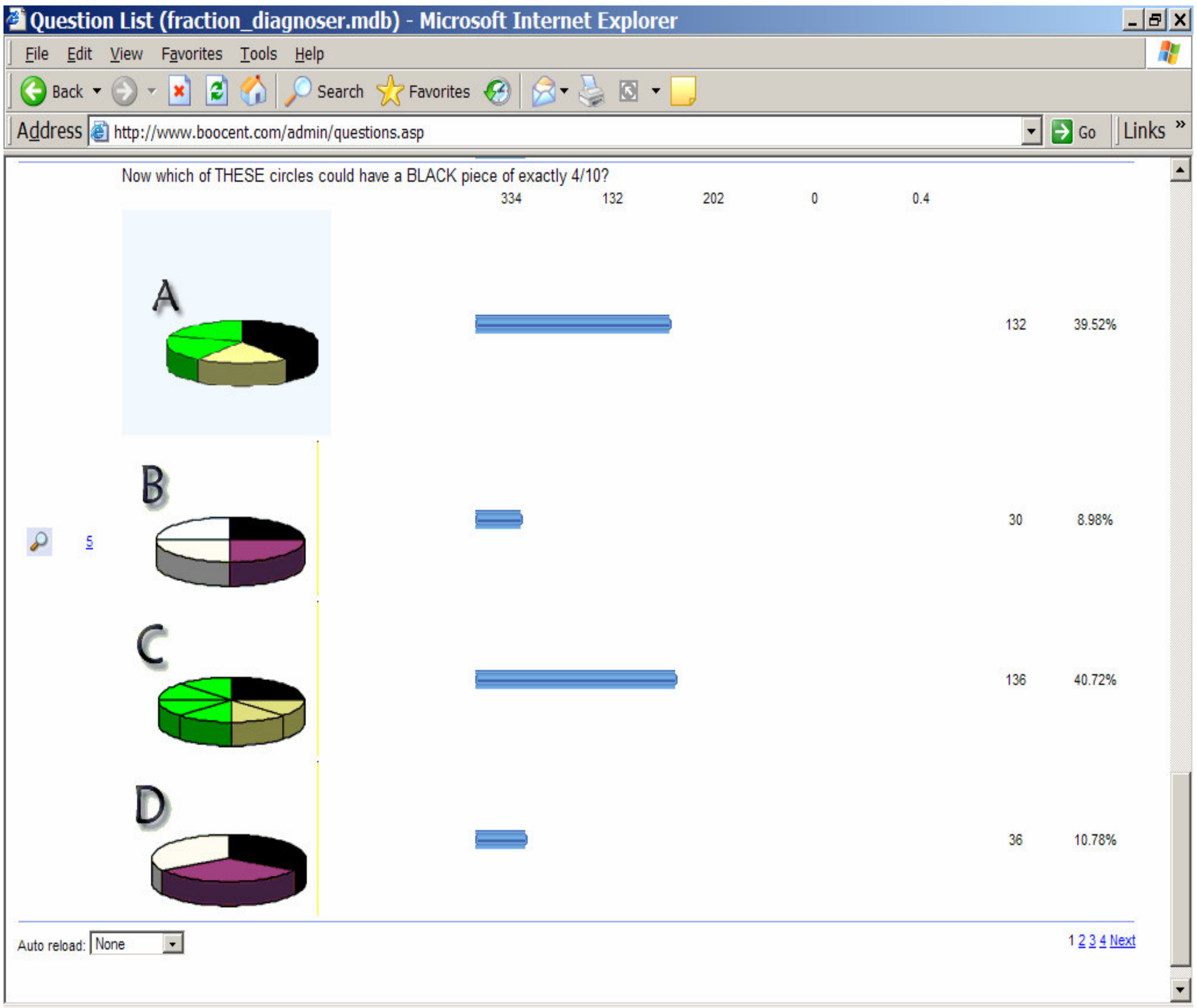


Figure 24. *Fraction Diagnoser* assessment item 5, showing 39.52% or 132 of the users choosing the correct response for question 5 (a).

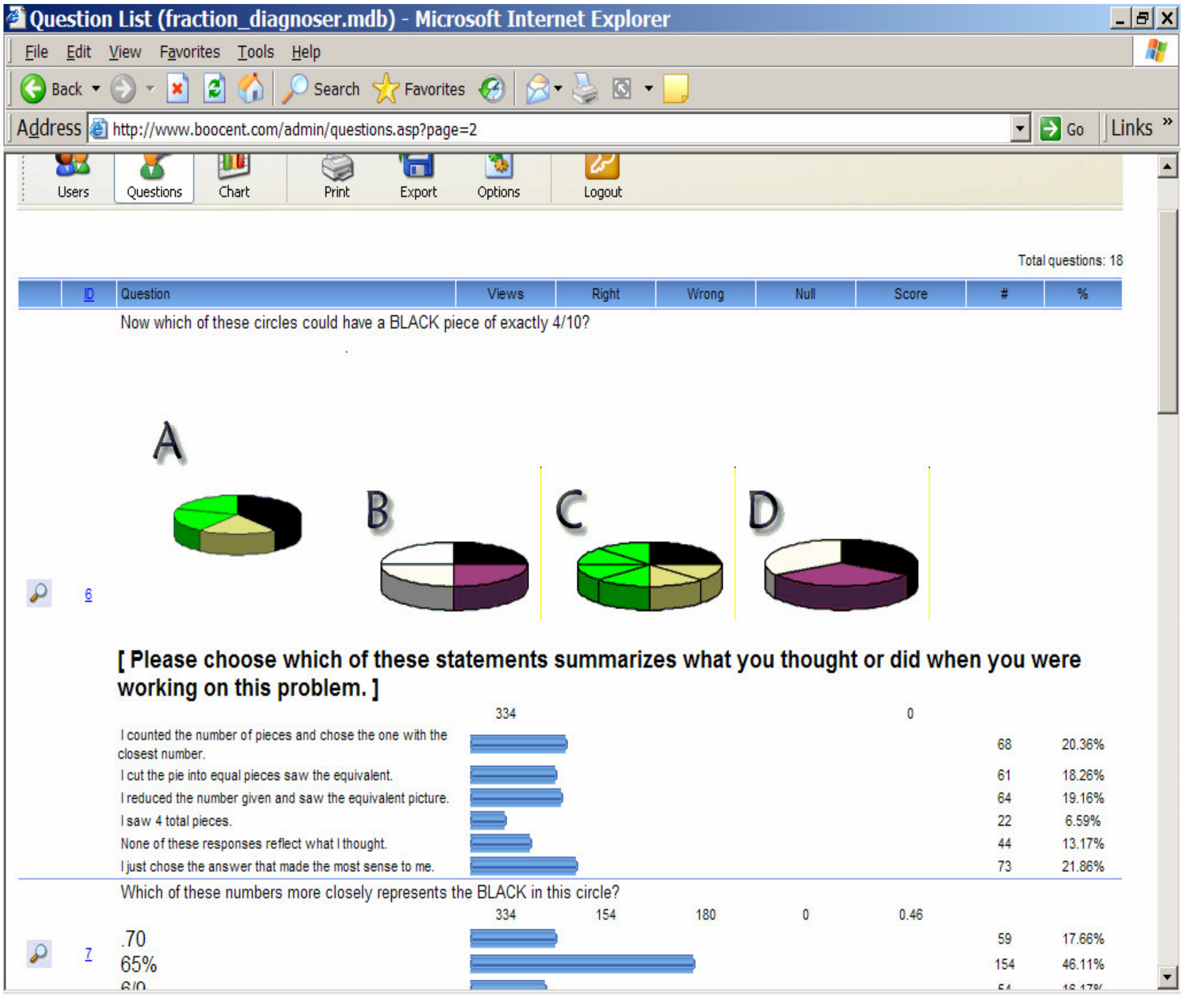


Figure 25. *Fraction Diagnoser* Question data showing 19.16% or 64 of the users choosing the *facet* that was scored highest for Diagnostic Item 6.

The data presented by the *Fraction Diagnoser* does document inconsistencies in student thinking, but further analyses of these findings go beyond the scope of this study. The research questions for this study focus on the reliability and validation of the instrument. Therefore, the statistical analyses necessary to evaluate the inconsistencies were not pursued.

Research Question 4 – Implementation: How Could Teachers Use the Information Provided by Fraction Diagnoser to Make Instructional Decisions?

Responses to Interview Questions

The seven teacher participants in this study were required to administer the *Fraction Diagnoser* to the students in their Math 0100, 0200, and 0300 classes. The teachers all administered the instrument as part of their normal course load throughout different times during the spring semester of 2005. With a range of 4 to 29 years of experience, the teachers represent over 70 years of teaching experience, with an average of 10.86 years per teacher. After 334 of their students had taken *Fraction Diagnoser*, the teachers were shown the summative and formative output data from the *Fraction Diagnoser* Database. They then responded to questions from the interview guide in Chapter 3 (see Table 5). Table 14 summarizes their responses to the questions:

Table 14. Presentation of Interview Data

Questions involving ratings (N = 7)	Mean	Standard Deviation
1. On a scale from 1-10, with 1 being completely useless and 10 being extremely useful, how useful do you think the *Fraction Diagnoser* results are?	8.71	0.76
3. On a scale from 1-10, with 1 representing extremely difficult and 10 representing effortless ease, how would you rate the *Fraction Diagnoser* in terms of difficulty of use for the student?	9.14	1.46
5. On a scale from 1-10, with 1 representing extremely difficult and 10 representing effortless ease, how would you rate the *Fraction Diagnoser* in terms of difficulty of use for you (teacher participant)?	9.57	0.79
8. On a scale from 1-10, with 1 representing "needing major changes" and 10 being "needing no improvement," how would you rate the *Fraction Diagnoser* in terms of effective content for your subject area?	9.07	1.10

Presentation of Elaboration Data

After each question listed above the teachers responded to the question, "Would you care to elaborate?" What follows is a summary of the repeated responses by participants and the number of participants who spoke the phrases.

Note that these are repeated responses. Only phrases heard more than once are summarized here:

1. (Elaborations) – needed more variety of items (4)
   - get a chance to see what students were thinking (5)
3. (Elaborations) – online and easily accessible (6)
   - could have been done at home (3)
5. (Elaborations) – very easy to use (7)
   - good assignment for the beginning of a unit (6)
   - can easily see individual student and overall class info (7)
   - could be used anytime in the semester (3)
8. (Elaborations) - needs more content than just fractions (6)

In addition to the responses to the elaboration responses the teachers were also asked a couple of other questions to address the research questions. The questions and responses were as follows:

7. In your expert opinion, what are the three major content areas of difficulty for students in developmental mathematics?

- word problems (7 participants)
    - fractions (7 participants)
- multi-step equations (4 participants)
- combining integers (3 participants)

10. How could you use the *Fraction Diagnoser* results information (show results) to make instructional decisions?

- design curriculum to address misconceptions (4 participants)
- use in the beginning to find out where students are (5 participants)
- could be done with each objective (6 participants)
- information about how they think (7 participants)

In addition to the questions on the research guide there were other comments made by teachers. These comments were suggestions related to instrument revision.

Summary of Results and Findings

The R&D cycle encompasses the research questions in Chapter Three, and in this chapter the findings from the research questions feed back into the R&D cycle. This chapter explains how the data collected in the steps in the Borg and Gall (1996) R&D cycle to design *Fraction Diagnoser* were used to address each research question. As written, each research question is addressed and the steps in the R&D cycle are informed from the findings that address the appropriate questions.

CHAPTER V

SUMMARY AND CONCLUSIONS

This R&D was done to produce *Fraction Diagnoser*, an alternative assessment instrument that assesses both student thought and understanding of MUL in developmental mathematics. *Fraction Diagnoser* is an online program administered via the web and used by participants on a personal computer. *Fraction Diagnoser* consists of sets of multiple-choice items that first assess student skills, then assess aspects of student thinking or reasoning related to MUL, which are called facets.

Understanding the complexity of mathematics requires a focus on the right concept. Minstrell's (2002) research found that to model the complexity of teaching and learning in the classroom, one needs to use the right level of description for that purpose. The description of students' thinking needed to be understood by teachers, by scientists and by researchers on learning; then this description should lead to a level that will serve classroom teachers as they make instructional decisions.

Summary of Procedures

The study followed the procedures of Borg and Gall (1996) for their R&D cycle. Those procedures were: (1) Research and collection of background information, (2) Planning the procedure of the study, (3) Preliminary product development, (4) Preliminary product test, (5) Product revision, (6) Main field test, (7) Operational product revision, (8) Operational field test, (9) Final Product Revision, and (10) Dissemination and implementation. All but Step 9 was used during the development of *Fraction Diagnoser*. Borg and Gall (1996) recommend that steps be regulated in a study where resources are limited, such as a dissertation.

Also, use of the 1996 Borg and Gall R&D cycle was much more useful than the 1989 version. The 1989 version presented a more linear approach to R&D which did not fit the model for *facet* collection very well (Minstrell, 2000). It is for this reason that any replications of this study should follow the 1996 version of Borg and Gall or a similar R&D model that allows for overlap of steps.

Reviewing the timeline for the study, the researcher also recommends that others looking to replicate be aware that Step 1 of the R&D cycle (research and collection of background information) could possibly overlap over half the study in this type of *facet* research. New *facets* could be found throughout the Borg and Gall (1996) R&D cycle, and although they would be considered a Step 1 process, they should not be dismissed due to timing. This type of *facet* research is evolutionary, and, to support that fact, Minstrell (1989, 2000, 2001, 2002) consistently points out that the collection of *facets* is an ongoing process.

Summary of Findings and Discussion

Summary of Findings and Discussion for Research Question 1: What Is the Facet Cluster Related to Multiple Meanings and Models of Fractions (MUL)?

The first steps in the development process indicated that a clear definition of learning goals and a comprehensible explanation of *facets* needed to be expressed and explained in order to alleviate dramatic changes in a *facet cluster*. Data collected in Step 3 of the R&D cycle indicated that the *a priori* table of *facets* first developed did not coincide very closely with what students were doing or thinking. In fact, the table looked like a grading rubric to assess only written skills related to MUL. The final table was revised to be more of what a MUL *facet cluster* should be (Minstrell, 2000), in terms of a measurement of student thinking.

Clearly, as the understanding of the MUL *facets* grew, the clarity of the MUL *facet* cluster began to materialize. Minstrell (2002) explained *facets* as descriptions of students' thinking created from what students say or do in the classroom or other learning situation. Therefore, a revised *facet* cluster should provide more information regarding students' thoughts. Also, the premise that *facets* of student thought can be seen as pieces of knowledge and/or strategies of reasoning defines the scope by which *facets* should be collected. When this definition is understood, collection and organization of *facets* are easier. Conclusively, one can see that the middle version of the MUL *facet* cluster expressed more relationship to Minstrell's definition of *facets*, while the final version of the MUL cluster was even better.

Facets are students' thoughts or strategies. Therefore, although a group of experts can theorize about what students' responses might be, collecting *facets* created from in-depth interviews is a far more effective method with which to find the *facet* cluster of interest. These findings can be compared to Minstrell's (2002, 2000) design, where he also concluded that *facet* clusters evolve toward more clarity. In fact, Minstrell (2002) stated that the identification of *facets* and *facet* clusters is an on-going, iterative research and development process.

The MUL *facet* cluster's inclusion of more than one *facet* cluster was one of the major findings. Clearly, MUL encompassed too many objectives or skills to be considered a single *facet* cluster. The fact that there were six general skills in the skills checklist should have been one indication that the subject matter selected was too broad. It became more apparent in Step 3 of the R&D cycle (preliminary product development) when questions were made to cover skills. Obviously, those skills would combine into different *facet* clusters. To alleviate these types of problems, Minstrell's (2002) *facet* research employed a much more defined objective when building *facet* clusters. For this reason, MUL would not be considered a good topic or focus for developing one *facet* cluster.

In addition, by comparing this study's findings to those of Minstrell (2000), *Fraction Diagnoser* could yield several *facet* clusters that would cover the MUL far more specifically. For example, using the final *facet* table as a reference, MUL could be the beginnings of a set of far more specific *facet* clusters:

Beginning of Un-named *Facet* Cluster for problem number 2 or equivalents.	020 - I counted each black equal part and used a calculator to convert the fraction to other numbers. 024 – I connected the two black areas in my mind and made comparisons without using a calculator. 025 - I made visual comparisons of the overall black area and just saw what the answer was. 027 - I counted each black part over the whole and including the hole in the middle.
Beginning of Un-named *Facet* Cluster for problem number 4 or equivalents.	040 - I counted the black part and made it a fraction of the whole. 043 – I used process of elimination, because the other answers don't make sense. 046 - I just thought it was common sense.
Beginning of Un-named *Facet* Cluster for problem number 6 or equivalents.	060 - I reduced the number given and saw the equivalent picture. 061 - I cut the pie into equal pieces and saw the equivalent. 067 - I saw 4 total pieces (numerator dependent). 067 - I counted the number of pieces and chose the one with the closest number (denominator dependent).
Beginning of Un-named *Facet* Cluster for problem number 8 or equivalents.	080 - I counted each black equal part and used a calculator to convert the fraction to other numbers. 082 - I calculated each piece and then added all of the black pieces together. 084 – I connected the two black areas in my mind and made comparisons without using a calculator. 085 - I made visual comparisons of the overall black area and just saw what the answer was.
Beginning of Un-named *Facet* Cluster for problem number 10 or equivalents.	101 - I thought any 3-D figure has volume. 105 - I thought they all had volume. 108 - I thought none of them had volume. 108 - I thought circles had volume.
Beginning of Un-named *Facet*	140 - I used a calculator to make fractions into decimals and compared.

Cluster for problem number 14 or equivalents.	140 - I found the common denominator on all of the fractions and compared 146 - I saw the numbers rising and knew that the numbers were getting bigger (or smaller).
Beginning of Un-named *Facet* Cluster for problem number 16 or equivalents.	161 - I found the fraction for that arrow and saw they were different. 162 - I looked at the number line and saw that the arrow was less than where it should be. 166 - I thought that the fraction can be on a number line, but not this one. 167 - I saw the arrow at the same number as the numerator. 169 - I thought there wasn't enough information to answer the question.
Facets for all problems	008 - None of these responses reflect what I thought. 009 - I just chose the answer that made the most sense to me.

By using each problem's *facets* and identifying the specific objective covered by the item, a researcher can begin to create the *facet* cluster for that learning objective.

Summary of Findings and Discussion for Research Question 2: (Between Subjects Validation) How Well Did the Fraction Diagnoser Identify Distinct Levels of Understanding of MUL Concepts and Skills for Individual Students in Developmental Mathematics?

A reliability coefficient as high as .82 was found for *Fraction Diagnoser* in its final form, and other statistical tests were done in order to validate the instrument. Specifically, both convergent and discriminant validity were found to be consistent with the theoretical designs of the instrument (Huck, 2000). *Fraction Diagnoser'*s scores from individual students along with their THEA scores for the number subgroup were used to validate.

Other findings indicated that the instrument was very reliable and valid, but the THEA number subgroup was not a good criterion variable. *Fraction Diagnoser* yielded a multiple R-squared of approximately 13%, and although this value would be considered as evidence of a good predictor, concerns still exist with the THEA number subgroup assessment.

The THEA does not provide adequate information as to student thought. As with most large scale summative assessment, the THEA separates student scores according to skills in subgroups. However, there is no understanding of student thought provided. Students with exact grades according to the THEA could exhibit extremely different *facets* in *Fraction Diagnoser,* and mathematics educators would agree that the thought process is just as important as a skill score.

Summary of Findings and Discussion for Research Question 3: (Within Subjects Validation) What Kinds of Student Information Did the Fraction Diagnoser Provide to Describe Student Growth Toward Mastery in MUL?

As seen from the figures, tables, and charts in Chapter IV, *Fraction Diagnoser* provides considerable information about students' understandings whenever they use the instrument. Individual data can be seen from the online program database, and teachers/researchers can use the information to diagnose or track individual student progress. *Fraction Diagnoser* data can also be exported to a database or spreadsheet. There, the statistical inferences are limited by the imagination of the researcher.

Information given by *Fraction Diagnoser* also adds to the wealth of cognitive research. This research provides more information on the thought processes of developmental mathematics students on the college level. And although the research done by the Rational Number Project (RNP) completely covers student reasoning and thoughts in the middle school levels, this *facet* research adds empirical data as to these developmental college students' thoughts. Therefore, the *facets* found by this research not only further enhance cognitive research by studying comparisons to other levels but also provide information to study individual student patterns.

Fraction Diagnoser provides the information necessary to track individual student information. If the interface of *Fraction Diagnoser* does not satisfy the researchers or teachers, they can download the data to a spreadsheet or database to provide graphs of student progress (see Figure 26).

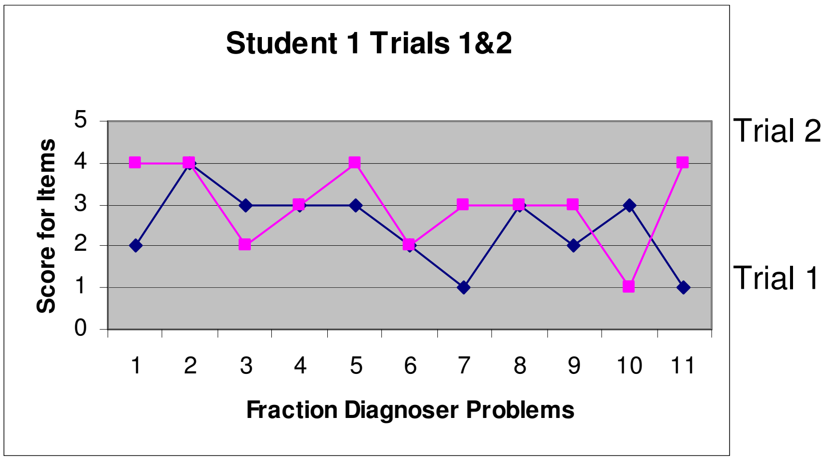


Figure 26. Example of graph of individual student responses to *Fraction Diagnoser* in trials 1 & 2.

Summary of Findings and Discussion for Research Question 4: (Implementation) How Could Teachers Use the Information Provided by Fraction Diagnoser to Make Instructional Decisions?

Interview data showed that the *Fraction Diagnoser* could be a useful instrument in developmental mathematics. The responses from the teacher participants indicated that the instrument was not only efficient but also effective in providing information on the student's thought processes. The data also supported the expansion of *Fraction Diagnoser* to include other mathematical concepts. The responses and comments found here are similar to the data found by Minstrell (2002) in his studies using *Diagnoser*.

Teachers made comments in addition to their responses to the questions on the research guide. These comments suggested ways to improve the development of the instrument. Two teacher participants pointed out that the diagrams of the figures could be made a little clearer where in some cases they might cause confusion. These teacher participants specifically pointed out a question in the *Fraction Diagnoser* where the lines did not separate the black parts to show equal segments. This was initially an issue with the test designers, but they went on to say that for this level of student that that should not be an issue.

Recommendations for Future Research

Recommendations for future research include the investigation of *facets* for different concepts throughout mathematics. *Facet* study is a very applicable type of research that involves the teachers. It was evident from this study that teachers have preconceived ideas of what students are thinking and that they were fascinated to learn the many different ways that students can think about a situation or subject. The finding and presentation of *facets* could help enlighten teachers to possible misconceptions, thereby improving instruction and learning. One next step for this research would be to determine the impact on student learning in classes in which teachers use feedback from the *Fraction Diagnoser* to design instruction.

Theoretically, as teachers are experts in content areas, they sometimes lose the ability to see the many wrong ways students think about content objectives. Closed-minded teaching is a liability in the classroom. If the teacher is shown the many ways students can think, and then is given different methods by which to address these issues, everyone benefits. It is the responsibility of the researcher to provide these types of “prescriptions” for teachers.

In addition, *Fraction Diagnoser* or similar instruments should be applied to different grade levels and different content areas. Eventually, a collection of these *facets* could provide a database for a large scale assessment instrument of this type. Correlations could be made across grade levels in MUL as to which *facets* were similar and problematic. Consequently, the data could provide evidence as to the effectiveness of the curriculum being taught at the school, district, or state levels.

Further Revisions and Implications

What *Fraction Diagnoser* offers is an alternative assessment that is not focused on a score, but provides enlightenment for teachers when evaluating student work as to the student’s thought processes. It is not uncommon for a teacher to want to know, “What were they thinking?” and *Fraction Diagnoser* may shed some light on that question. Also this research can be used to provide online help for students, much like the original *Diagnoser*, that assists students' metacognition. This type of diagnostic assessment can give more specifics to both the student and the teacher as to where the student's understanding is at a particular time so that a teacher knows what needs to be addressed in instruction (Minstrell, 2002).

Also, an instrument like this *Fraction Diagnoser* could be used in a large scale assessment and provide more information than the current assessments. As previously stated, the THEA is very inadequate in providing information concerning the thought processes of students. Mathematics educators want information concerning student thought--not just the summarized version of what researchers have presented, but collections of raw data that provide as close to what the student is saying as possible.

Student thought needs to be properly documented. *Fraction Diagnoser* and similar instruments could provide this type of documentation. Most of the studies previously referenced summarized and presented the student data. This should make you wonder how much of the student's true words were lost in the translation. The more processed the explanation of what a student actually thought, the further you get from understanding what the student is actually thinking. The data provided by the Rational Number Project (RNP) was immense but many times translated. *Fraction Diagnoser* and similar instruments should look to provide less translation and more directly accessed *facets*.

Fraction Diagnoser could benefit from providing a more detailed facet profile of student responses. Although the interface appeared adequate to the teachers in Step 10 of the R&D cycle, *Fraction Diagnoser* could be modified to gather together the *facet* profiles of students and present those in a form more effective for researchers. For example, this change in the instrument could allow researchers to focus on the cognitive diagnostic part in a form that could allow for efficient comparisons to other cognitive studies of this type.

In addition, *Fraction Diagnoser* could be modified to be in a more non-linear format. Each student responded to each question in the instrument, and this was effective. But the original *Diagnoser* was built with a program that allowed for more responsive questioning. Time constraints and financing limited this modification for *Fraction Diagnoser* for this study. If *Fraction Diagnoser* was modified to imitate this format, the instrument could provide a branching profile of *facets* for each student.

Conclusions

This chapter summarizes the significance of the findings in this study. Each research question is individually addressed and elaborations are made concerning how they were addressed. A section is done that focuses on the possible further implications of this instrument and its uses in research. Also, this chapter elaborates on the revisions that could be made to make this study more complete for replications.

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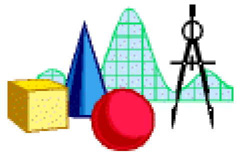
APPENDIX A

THEA MATHEMATICS PRACTICE TEST

MATHEMATICS SECTION



The mathematics section of the test consists of 48 multiple-choice questions. Read each question carefully and choose the ONE best answer.

Appropriate definitions and formulas are provided below to help you perform the calculations on the test.

Definitions and Formulas

Definitions

$=$	is equal to	$\perp$	is perpendicular to
$\neq$	is not equal to	$\parallel$	is parallel to
$\approx$	is approximately equal to	$\sim$	is similar to
$>$	is greater than	$\cong$	is congruent to
$<$	is less than	$\ncong$	is not congruent to
$\geq$	is greater than or equal to	$\pm$	plus or minus
$\leq$	is less than or equal to	$\overline{AB}$	line segment joining points A and B
π	≈ 3.14	$\overleftrightarrow{AB}$	line containing points A and B
$\angle$	angle	$m(\overline{AB})$	length of $\overline{AB}$
$m\angle$	measure of angle	AB	length of $\overline{AB}$
$\llcorner$	right angle	$\lvert\overline{AB}\rvert$	length of $\overline{AB}$
$\triangle$	triangle	$\frac{a}{b}$ or a:b	ratio of a to b

Abbreviations for Units of Measurement

		U.S. Standard		Metric
Distance	in.	Inch	m	meter
	ft.	Foot	km	kilometer
	mi.	Mile	cm	centimeter

			mm	millimeter
Volume	gal.	Gallon	L	liter
	qt.	Quart	mL	milliliter
	oz.	Ounce	cc	cubic centimeter
Weight/Mass	lb.	pound	g	gram
	oz.	ounce	kg	kilogram
			mg	milligram
Temperature	°F	degree Fahrenheit	°C	degree Celsius
Time	sec.	second		
	min.	minute		
	hr.	hour		
Speed	mph	miles per hour		

Conversions for Units of Measurement

	U.S. Standard		**Metric**
Length	12 inches = 1 foot	**Length**	10 millimeters = 1 centimeter
	3 feet = 1 yard		100 centimeters = 1 meter
	5280 feet = 1 mile		1000 meters = 1 kilometer
Volume	8 ounces = 1 cup	**Volume**	1000 milliliters = 1 liter
(liquid)	2 cups = 1 pint		1000 liters = 1 kiloliter
	2 pints = 1 quart		
	4 quarts = 1 gallon		
Weight	16 ounces = 1 pound	**Weight**	1000 milligrams = 1 gram
	2000 pounds = 1 ton		1000 grams = 1 kilogram

Time 60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day

Formulas

Quadratic formula: If $ax^2 + bx + c = 0$ **, and a** $\neq$ **0,**

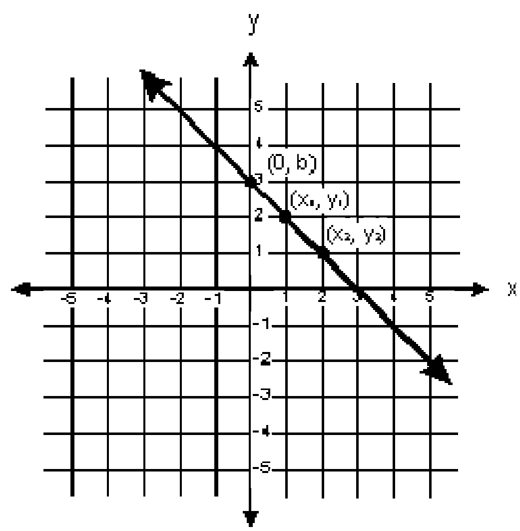
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Line

Slope = $m = \frac{y_2 - y_1}{x_2 - x_1}$

Slope-intercept form for the equation of a line
$y = mx + b$

Point-slope form for the equation of a line
$y - y_1 = m(x - x_1)$



Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

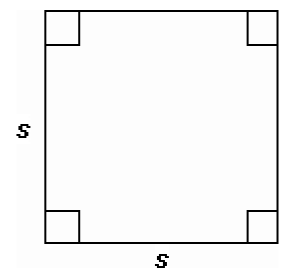
Distance

d = rt

Geometric Figures

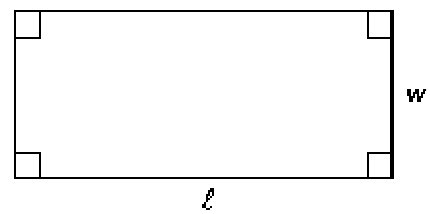
Square

Area = s^2
Perimeter = $4s$



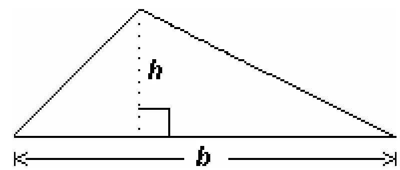
Rectangle

Area = ℓw
Perimeter = $2\ell + 2w$



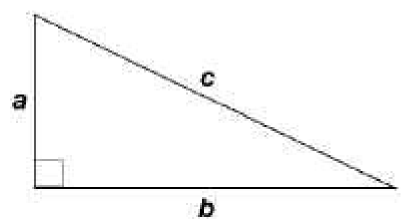
Triangle

Area = $\frac{1}{2}bh$



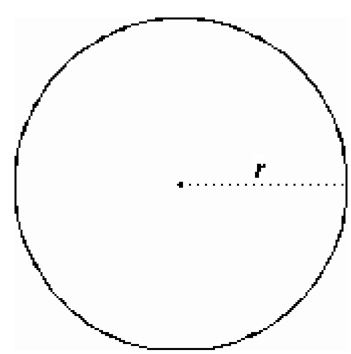
Right triangle

Pythagorean formula: $c^2 = a^2 + b^2$



Circle

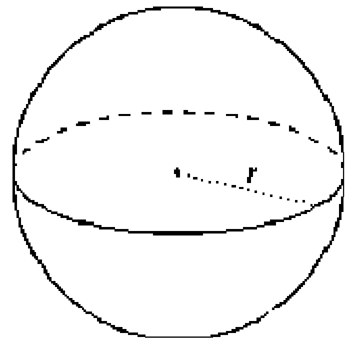
Area = πr^2
Circumference = $2\pi r$
Diameter = $2r$



Sphere

Surface area = $4\pi r^2$

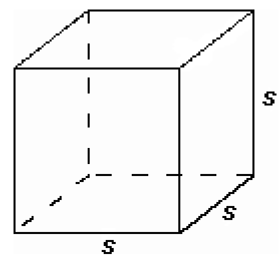
Volume = $\frac{4}{3}\pi r^3$



Cube

Surface area = $6s^2$

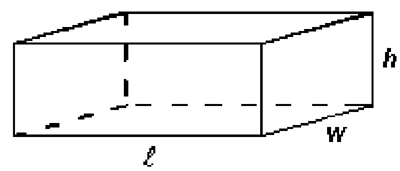
Volume = s^3



Rectangular solid

Surface area = $2\ell w + 2\ell h + 2wh$

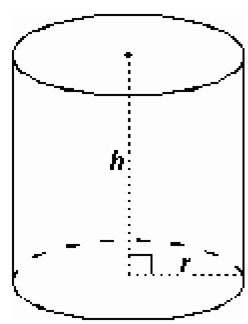
Volume = ℓwh



Right circular cylinder

Surface area = $2\pi rh + 2\pi r^2$

Volume = $\pi r^2 h$



1. A machine in a soft drink bottling factory caps 3 bottles per second. How many bottles can it cap in 15 hours?

A. 2.7×10^3

B. 1.6×10^4

C. 1.8×10^4

D. 1.6×10^5

2. A truck has a full 50-gallon gas tank. It uses $7\frac{1}{4}$ gallons on the first part of its journey, $13\frac{1}{2}$ gallons on the second part of its journey, and $15\frac{1}{4}$ gallons on the third part of its journey. How many gallons of gas remain in the gas tank?

A. 14

B. $14\frac{1}{4}$

C. 15

D. 36

3. A rancher is planning to put up 220 yards of fencing. In the morning she puts up 80 yards, and in the afternoon she puts up 40% of the remaining fence. What percent of the fence did she put up that day?

A. 36%

B. 51%

C. 62%

D. 76%

4. During a bike-a-thon a local company pledges to donate $1.25 for every $4.00 pledged by the public. If the public pledges a total of $156.00 dollars per mile, how much will the company donate per mile?

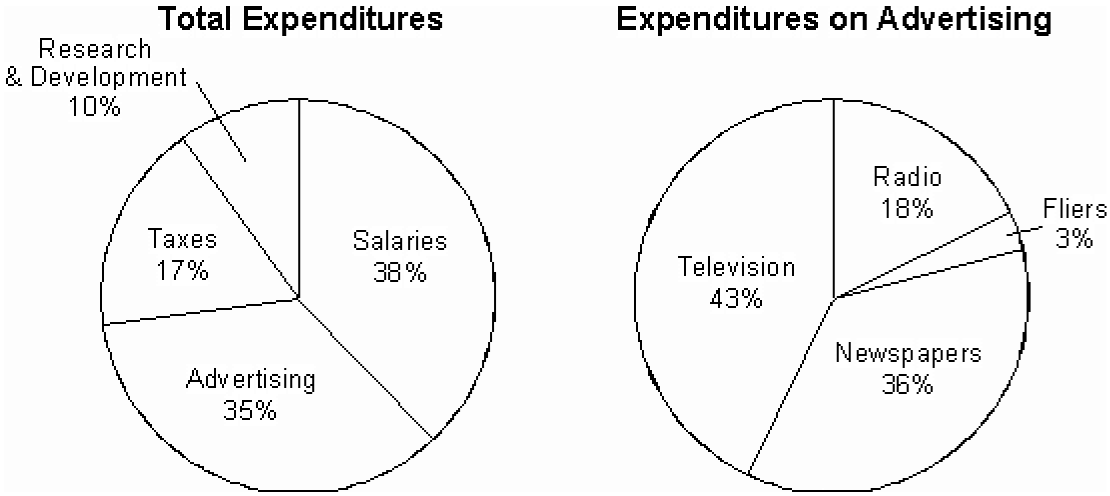
A. $2.75

B. $48.75

C. $195.00

D. $499.20

5. Use the pie charts below to answer the question that follows.



The first pie chart represents a company's expenditures, and the second pie chart shows a breakdown of the company's advertising expenditures. What percent of the company's expenditures is spent on radio advertising?

A. 6.3%

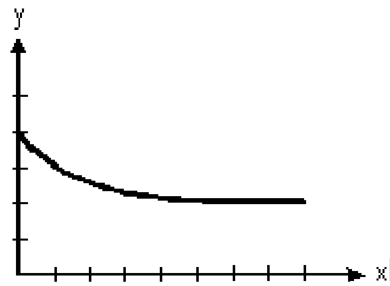
B. 11.7%

C. 18.0%

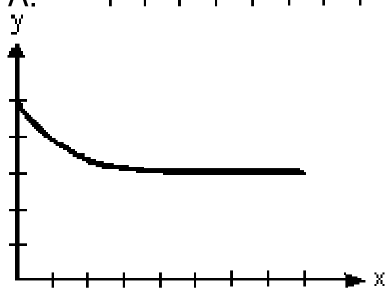
D. 35.0%

6. Scientists have stocked Wilson's pond with a species of fish. The scientists note that the population has steadily decreased over a period of time until the population is approximately half the number of fish originally stocked. If the number of fish are plotted on the *y*-axis and the amount of time on the *x*-axis, which of the following could result?

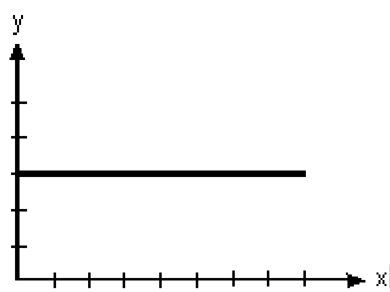
A.



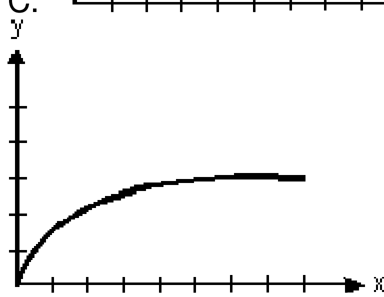
B.



C.



D.



7. A student has received scores of 88, 82, and 84 on three quizzes. If tests count twice as much as quizzes, what is the lowest score the student can get on the next test to achieve an average score of at least 70?

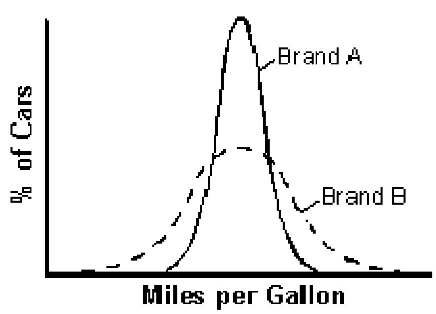
A. 13

B. 48

C. 70

D. 96

8. Use the distribution curves below to answer the question that follows.



The distribution curves above show data on the gas mileage for two different brands of car. Which of the following correctly analyzes the information presented in these distributions?

A. the mean gas mileage of brand *A* is greater than the mean gas mileage of brand *B*

B. data was collected for more cars of brand *A* than of brand *B*

C. brand A cars have smaller variability in gas mileage than brand B cars

D. brand A cars get poorer gas mileage than brand B cars

APPENDIX B

INITIAL FRACTION DIAGNOSER – STEP 3 OF R&D CYCLE

Fraction Diagnoser

Class ACCESS

Enter name

Answer each question by selecting the correct button.

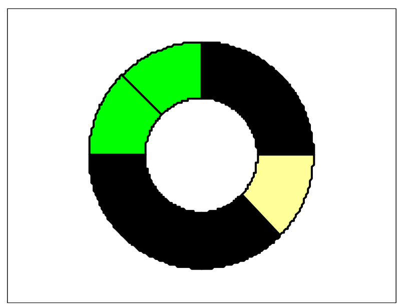
1. Which of these numbers more closely represents the BLACK area in this figure?

☐ A. .70

☐ B. 65%

☐ C. 6/9

☐ D. 3/4



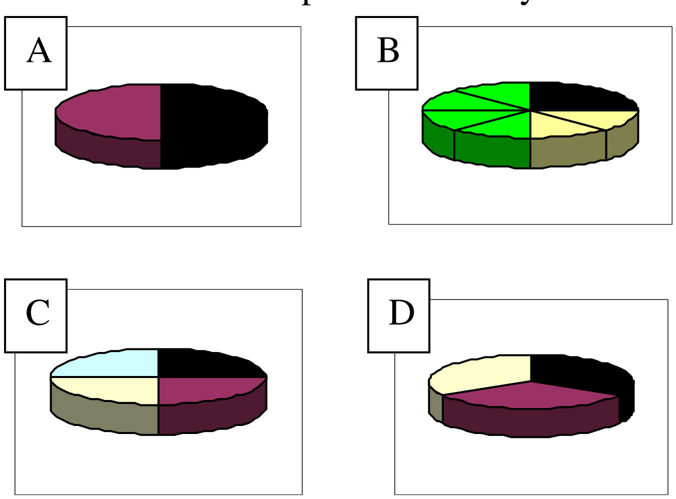
2. Which of these circles has a BLACK piece of exactly 1/3?

☐ A.

☐ B.

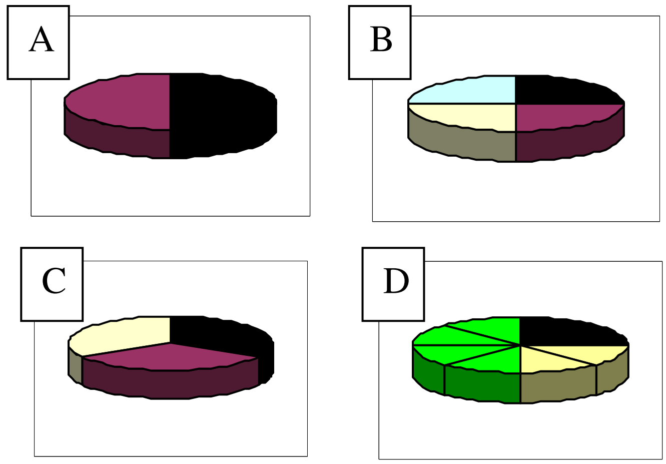
☐ C.

☐ D.



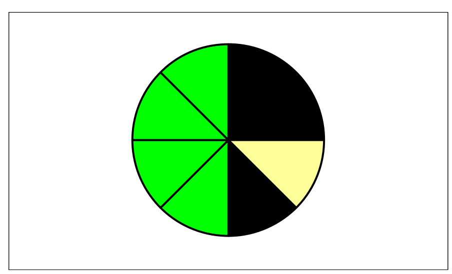
3. Which of THESE circles could have a BLACK piece of exactly 4/12?

☐ A.
☐ B.
☐ C.
☐ D.



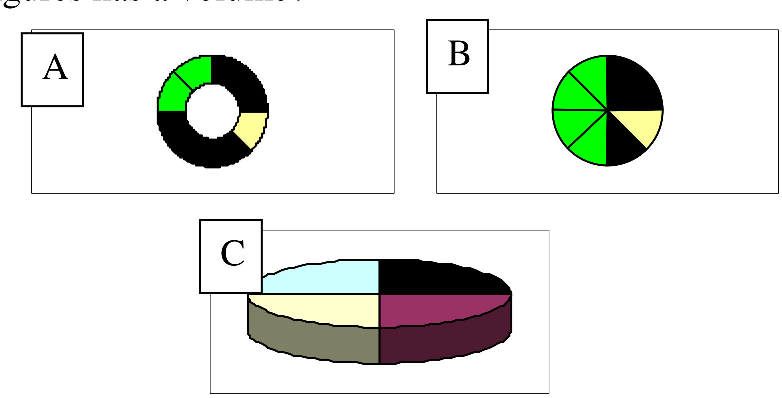
4. Which of these numbers more closely represents the BLACK in this circle?

☐ A. 3/10
☐ B. .50
☐ C. .40
☐ D. 1/3



5. Which of these figures has a volume?

☐ A.
☐ B.
☐ C.
☐ D. Neither



6. Are these fractions in order?

$$\frac{1}{3} \longrightarrow \frac{2}{5} \longrightarrow \frac{3}{15}$$

- ☐ A. Yes, descending (getting smaller)
- ☐ B. No
- ☐ C. Yes, ascending (getting larger)
- ☐ D. They are equal

7. Which of those fractions from problem # 6 is the largest?

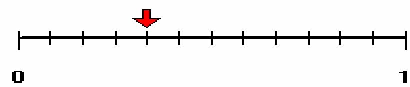
- ☐ A. 2/5
- ☐ B. 1/3
- ☐ C. 3/15
- ☐ D. Neither

8. Which of those fractions from problem #6 is the smallest?

- ☐ A. 2/5
- ☐ B. 1/3
- ☐ C. 3/15
- ☐ D. Neither

1. Does the arrow below point at 4/6 on this number line?

Identify Fractions With Lines



- A. No
- B. Yes
- C. The fraction 4/6 can ONLY be represented on a pie chart/graph
- D. The fraction 4/6 is no where on this number line

Grade Exam

Note: When "Grade" is chosen, answers cannot be changed. Only unanswered questions can be selected.
When "Submit" is chosen no answer can be changed.

APPENDIX C

SKILLS CHECKLIST FOR FRACTION DIAGNOSER ASSESSEMENT ITEMS SCORING

Student sees equal sections 1/b
Student identifies visual fractions effectively (visual to fraction)
Student successfully compared a percent to a decimal
Student successfully compared a fraction to a percent
Student can find ratio 1/b in a/b
Student identifies visual fractions effectively (fraction to visual)
Student can identify 3-D figure
Student can recognize fractions as area models
Student found common denominators
Student compared numerators
Student converted fraction to decimal
Student successfully compared decimals
Convert fractions to decimals
Student successfully compared fraction to a decimal
Student can count equal subsections to equal total fraction
Count sections up to arrow
Student found ratio a/b
Student can reduce ratio to lowest terms

** Each skill was worth one point for skills questions

The above checklist skills are expanded from the following learning objectives:

1. Student can compare decimals, fractions, and percents.
2. Student can show multiplicative reasoning when looking for equivalency.
3. Student can determine that in the fraction *a/b*, 1/*b* represents equal sections.
4. Student can convert *a/b* to a decimal by dividing *a* by *b*.
5. Student can see rational numbers in models of area or measurement.
6. Student can see a model being more useful than another, depending on purpose.

APPENDIX D

FRACTION DIAGNOSER'S RUBRIC FOR DIAGNOSTIC ITEM SCORING

Demonstrated Competence

Exemplary response (6 points) – Student giving a complete response with a clear, coherent, unambiguous, and elegant explanation; this response may include a clear and simplified diagram, communicated effectively to the identified audience, showing understanding of the assessment items' mathematical ideas and processes, identified all important elements of the problem, may include examples and counter examples, presents strong supportive arguments

Component response (5 points) – Gives fairly complete response, fairly clear explanations, includes an appropriate diagram, communicates effectively, shows understanding of the problem's mathematical ideas and processes, identifies the most important elements of the problem, presents a solid argument

Satisfactory Response

Minor flaws (4 points) – Satisfactorily completes the problem, a muddled explanation, incomplete argumentation, diagram unclear or inappropriate, understands underlying mathematical ideas, uses mathematical ideas effectively

Serious flaws (3 points) – Began problem appropriately, failed to complete it, omitted significant parts, failed to show full understanding of mathematical ideas and processes, major computational errors, misuse or lack of use of mathematical terms, used an inappropriate strategy

Inadequate Response

Begins but fails to complete problem (2 points) – Cannot understand explanation, unclear diagram, shows no understanding of the problem situation, major computational errors

Unable to begin (1 point) – Inappropriate explanation, diagram misrepresents the problem, copies problem but no attempt at a solution, fails to identify appropriate information

No attempt (0 points)

Adapted from the California Mathematics Council's Rubric for Open-Ended Questions.

APPENDIX E

ANSWER KEY TO FINAL FRACTION DIAGNOSER

Question 1	
5 PTS.	See equal sections
	Find how many equal sections are black
	Find total of equal sections
	Find ratio of black
	Convert to decimal
Question 2	
A	5 points
B	6 points
C	4 points
D	3 points
E	2 points
F	1 point
Question 3	
3 PTS.	Find total equal sections
	How many shaded black
	Make equal ratio
Question 4	
A	6 points
B	3 points
C	4 points
D	2 points
E	1 point
Question 5	
4 PTS.	Create equal sections (reduce)
	Find how many equal sections are black
	Make ratio
	See equivalence to fraction
Question 6	
A	3 points
B	5 points
C	6 points
D	3 points
E	2 points
F	1 point
Question 7	
4 PTS.	See equal sections
	Determine black sections
	Make ratio
	Convert effectively

Question 8	
A	3 points
B	5 points
C	4 points
D	4 points
E	2 points
F	1 point
Question 9	
1 PT.	Identify 3-D figure
Question 10	
A	3 points
B	1 point
C	6 points
D	1 point
E	2 points
F	1 point
Question 11	
3 PTS.	Recognize fractions
	Common denominators
	Compare numerators
	OR
	Convert to decimal
	Compare decimals
Question 12	
2 PTS.	Convert fractions to decimals
	Compare
Question 13	
2 PTS.	Convert fractions to decimals
	Compare
Question 14	
A	6 points
B	3 points
C	6 points
D	2 points
E	1 point
Question 15	
5 PTS.	See/count equal subsections
	Count sections up to arrow
	Find ratio
	Reduce ratio to lowest terms
	Compare to given fraction
Question 16	
A	4 points
B	3 points

C	5 points
D	6 points
E	1 point
F	2 points
Question 17	
1 PT.	Identify arrow 1 < X < 2
Question 18	
1 PT.	Compare decimals

** Answer key components from skills checklist are abbreviated for Assessment Items.

VITA

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